For #1 and #2, <u>find</u> and <u>classify</u> the critical point of each function.

1. $f(x,y) = 4x^2 + 2y^2 - 2xy - 10y - 2x + 1$

Finding the critical point:

a. Take the partial derivative of f with respect to x and set it equal to zero.

$f_x = 8x - 2y - 2 \rightarrow 8x - 2y - 2 = 0$

b. Take the partial derivative of f with respect to y and set it equal to zero

$f_y = 4y - 2x - 10 \rightarrow 4y - 2x - 10 = 0$

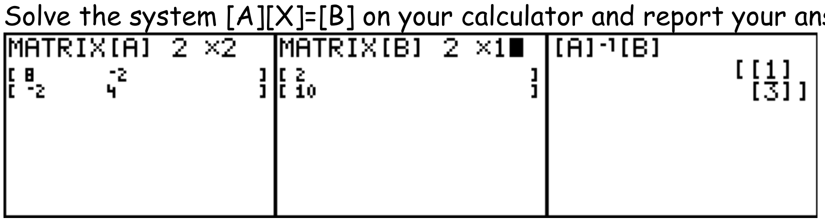
c. Set up a system of equations, rewriting them so you can use matrices.

$$8x - 2y = 2$$
$$-2x + 4y = 10$$

d. Write the [A], [X], and [B] matrices.

$$[A] = \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix} \qquad [X] = \begin{bmatrix} x \\ y \end{bmatrix} \qquad [B] = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

e. Solve the system [A][X]=[B] on your calculator and report your answers.



$$[X] = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

f. Find the output value at the critical point. $f(1,3) = -15$

Classifying the critical point:

g. Find the second partials of f(x,y). Write the second partials matrix.

$f_{xx} = 8 \qquad f_{xy} = -2$

$f_{yx} = -2 \qquad f_{yy} = 4$

$$\begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix}$$

h. Find the value of the determinant of the second partials matrix you found in part g.

$$\begin{vmatrix} 8 & -2 \\ -2 & 4 \end{vmatrix} = (8)(4) - (-2)(-2) = 28$$

i. Use the determinant test to classify the optimal point. Specifically state how you decided on the type of point.

$f(1,3) = -15$ is a relative minimum because $D(1,3) = 28 > 0$ and $f_{xx}(1,3) = 8 > 0$

2. Find and classify the critical point of f(x,y) = $2x^2 + 2xy + 6x + 4y + 10$

Find the 1st partials

$f_x = 4x + 2y + 6$

$f_y = 2x + 4$

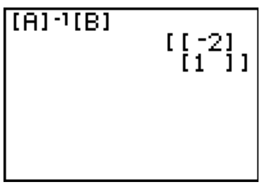
Set them = 0 and rewrite

$$\begin{matrix} 4x + 2y + 6 = 0 \\ 2x + 4 = 0 \end{matrix} \rightarrow \begin{matrix} 4x + 2y = -6 \\ 2x + 0y = -4 \end{matrix}$$

Set up the system of matrices

$$[A] = \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix} \qquad [X] = \begin{bmatrix} x \\ y \end{bmatrix} \qquad [B] = \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

Solve [X] =[A]$^{-1}$[B]



$$[X] = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Find the output at the critical point: $f(-2,1) = 6$

Find the 2nd partials matrix

$$f_{xx} = 4 \qquad f_{xy} = 2$$

$$f_{yx} = 2 \qquad f_{yy} = 0$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$$

Evaluate the determinant at the critical point

$$\begin{vmatrix} 4 & 2 \\ 2 & 0 \end{vmatrix} = (4)(0) - (2)(2) = -4$$

Classify the critical point using the determinant test

$f(-2,1) = 6$ is a saddle point because $D(-2,1) = -4 < 0$

3. The function $f(x,y) = 4xy - x^4 - y^4$ has three critical points, (0,0,0), (1,1,2) and (-1,-1,2).

Classify each point as a relative maximum, relative minimum or saddle point.

Hint: You should find the second partials matrix before you begin.

$f_x = 4y - 4x^3$ $\quad$ $f_y = 4x - 4y^3$

$f_{xx} = -12x^2$ $\quad$ $f_{xy} = 4$

$f_{yx} = 4$ $\quad$ $f_{yy} = -12y^2$

$$\begin{bmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{bmatrix}$$

Work for the point (0,0,0):

$$\left.\begin{vmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{vmatrix}\right|_{(0,0)} = \begin{vmatrix} 0 & 4 \\ 4 & 0 \end{vmatrix} = (0)(0) - (4)(4) = -16 < 0$$

The point (0,0,0) is a <u>saddle point</u>

Work for the point (1,1,2):

$$\left.\begin{vmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{vmatrix}\right|_{(1,1)} = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix} = (-12)(-12) - (4)(4) = 128 > 0 \text{ and } f_{xx}(1,1) = -12 < 0$$

The point (1,1,2) is a <u>relative maximum</u>

Work for the point (-1,-1,2):

$$\left.\begin{vmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{vmatrix}\right|_{(-1,-1)} = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix} = (-12)(-12) - (4)(4) = 128 > 0 \text{ and } f_{xx}(1,1) = -12 < 0$$

The point (-1,-1,2) is a <u>relative maximum</u>