

1. The yearly labor cost to produce lumber products at a large site is given by
 $C(m, h) = m^2 + hm + 3h^2 - 2m$ million dollars where m million \$ is the cost for administrative staff wages and h million \$ is the cost for hourly employee wages. [Check: $C(20, 20) = 1960$]

There is a constraint of $g(m, h) = m + 2h = 36$ thousand dollars on money spent recruiting labor for the company.

- a. Set up the system of 3 equations that would be used to find the optimal point under the constraint.

- b. Find the optimal point:

$m =$ _____, $h =$ _____, $C(m, h) =$ _____

- c. What is the nature of the optimal point? Complete the table below:

m	h	$g = m + 2h$	$c(m, h)$

The optimal point is a _____. Explain.

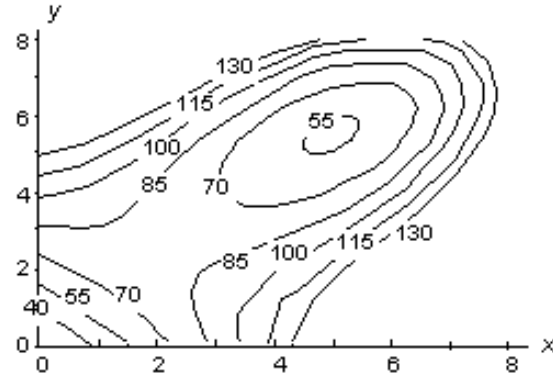
- d. Give the value of λ with units.
- e. If the constraint changes to 34, estimate the change in the critical point.
- f. If the constraint changes to 37, estimate the change in the critical point.

2. The number of silk plants (in thousands) that are sold by a chain of discount stores when the plant is $(x + 1)$ feet tall and the store carries y similar varieties of live plants can be modeled by $f(x, y)$.

An inventory and shipping constraint on the silk plants is given by the equation

$$g(x, y) = 8x + 7y = 56 \text{ feet of height}$$

- a. Graph the constraint $g(x, y)$ on the graph to the right that shows some of the contours of $f(x, y)$.



- b. List the outputs of the contours that the constraint g touches or crosses.
- c. Estimate the value of x , y and $f(x, y)$ at the optimal point (the point where the constraint is tangent to the contour). Hint: Your estimate must satisfy the constraint.
- d. Explain whether the optimal point is a maximum or minimum point.
- e. Interpret the optimal point.

3. In their spare time, Ben and Dennis are in a singing group called the Optimal Tones. The group has decided to make two recordings to sell, one called Maximum Fun (MF) and the other called Minimize Your Tears (MYT). The profit from the sale of these recordings can be modeled by $P(f, t) = 2000f - 100f^2 - 10ft + 500t - 10t^2$ dollars, where f is the number of MF recordings sold (in hundreds) and t is the number of MYT recordings sold (in hundreds).

Check that you have entered the equation correctly with $P(8,5) = 11450$.

Round answers based in the given units (CD's - units are hundreds, dollar - round to the nearest penny).

- a. If there are no restrictions on sales, determine the maximum profit and the corresponding number of MF and MYT recordings sold. [Hint: This is *unconstrained* optimization.]

$f =$ _____ $t =$ _____

$P(f,t) =$ _____ (Include units with all three)

- b. Find the second partials matrix.

- c. What is the value of the determinant at the optimal point?

We can verify the optimal is a maximum point because

1. _____ and 2. _____

- d. The MF recording is a two-disc set and the MYT recording is a 3-disc set. The production company has a supply of 100 hundred discs. Write this constraint as a mathematical equation.
- e. Subject to the constraint you found in (d), find the number of recordings that must be sold in order to maximize the profit. Set up all equations and matrices.

$$f = \underline{\hspace{10em}} \quad t = \underline{\hspace{10em}}$$

$$P(f,t) = \underline{\hspace{10em}} \quad (\text{Include units with all three})$$

- f. Verify that this point is a maximum using the close point method.

Type of point	f	t	P(f,t)
close point			
optimal point			
close point			

The output of P at each of the close points is _____ than the output of P at the optimal point.

g. $\lambda = \underline{\hspace{10em}}$ (include units)

- h. Interpret λ in the context of the problem.