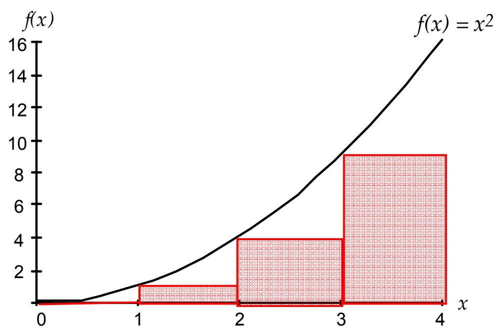
1. Complete the following table with outputs corresponding to the given inputs:

x	0	1	2	3	4
$f(x) = x^2$	0	1	4	9	16

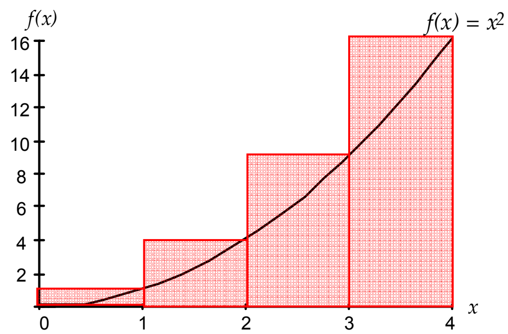
2. Estimate the area of the region between $f(x)$ and the input axis for x between 0 and 4 using each of the following approximation techniques. In each case, show your work and draw a figure to geometrically indicate the process.

4 left rectangles:



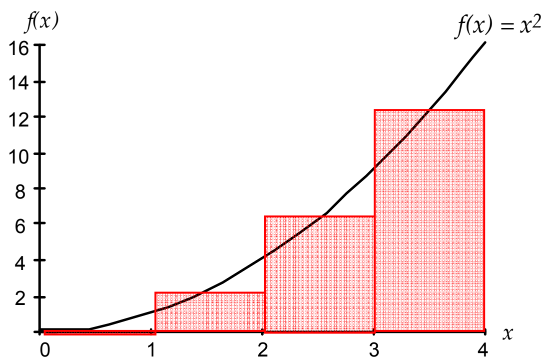
$$Area = 1[f(0) + f(1) + f(2) + f(3)] = 14$$

4 right rectangles:



$$Area = 1[f(1) + f(2) + f(3) + f(4)] = 30$$

4 midpoint rectangles:



$$Area = 1[f(0.5) + f(1.5) + f(2.5) + f(3.5)] = 21$$

3. Which method clearly over-estimates the area trapped between $f(x)$ and the x-axis?
   Right Rectangles

4. Which method clearly under-estimates the area trapped between $f(x)$ and the x-axis?
   Left Rectangles

3. The actual area between f and the x-axis from 0 to 4 is $21\frac{1}{3}$. Complete the following table:

Approximation Method	Area Approximation	Absolute Error = |True Value – Approx.|
Left Rectangles	14	$7\frac{1}{3}$
Right Rectangles	30	$8\frac{2}{3}$
Midpoint Rectangles	21	$\frac{1}{3}$

Which method gives the best approximation? Midpoint Rectangles

Which method(s) overestimates the area between f and the x-axis from 0 to 4? Right Rectangles

Which method(s) underestimates the area between f and the x-axis from 0 to 4? Left and Midpoint Rectangles

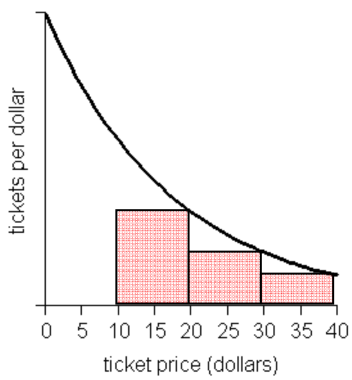
2. A small college is looking at data on the sale of tickets to their home football game (their undefeated, conference winning football team). The following data show the rate of change of the ticket sales at a given ticket price.

Ticket Price (dollars)	10	15	20	25	30	35	40
ROC of Sales (tickets/dollar)	183	138	103	80	58	45	33

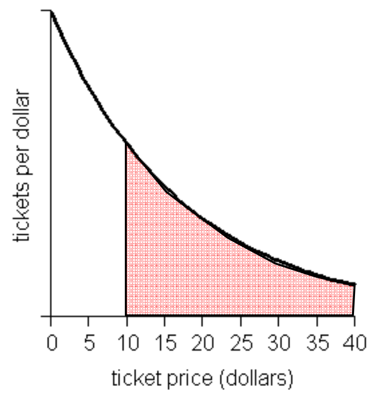
a. Fit an exponential model to the data. Report the model below. Be sure that your model is fully defined. (Review: This includes an equation, output units and description, input units and description and domain.)

$f(x) = 323.764(0.945^x)$ tickets per dollar gives the rate of change of ticket sales where x is the price of ticket in dollars, $10 \le x \le 40$.

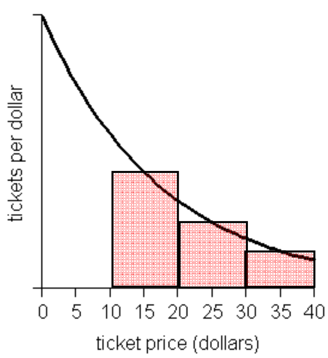
b. Draw 3 right rectangles that you would use to estimate the change in sales from $10 to $40. Do Not Find the Accumulation.

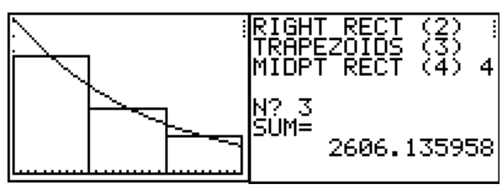


c. Shade the area that represents the exact change in sales when the ticket price increases from $10 to $40.



d. **Using the full model** (which should be pasted into your Y_1), use **three midpoint rectangles** to find the change in sales when the ticket price increases from $10 to $40. ***Sketch*** the three rectangles on the graph below. Write a sentence interpreting your result. Think about the units on your answer before writing your sentence and round the answer in the context of the problem.





Units: $\frac{tickets}{dollar} \cdot dollar = tickets$

Answer: 2606 tickets

e. Write a sentence of interpretation for your answer to part d.

When ticket prices increased from $10 to $40, the ticket sales increased by 2606 tickets.

3. Due to the rise in the cost for a college education, more and more students are attending in-state schools. The table below gives the rate of change in in-state applications for a given university.

End of School Year	1995	1996	1997	1998	1999	2000	2001	2002
Rate (hundred applications per year)	7.46	4.48	2.21	0.76	0.21	0.41	1.45	3.48

a. Align the input to the number of years after the end of the 1990 school year and find the best-fitting quadratic model for the data. Call the model *a(x)*. Define the model completely.

***$a(x) = 0.415x^2 - 7.646x + 35.349$* hundred applications per year gives the rate of change in the number of in-state applications, *x* years after the end of the 1990 school year, $5 \le x \le 12$.**

b. If you need to approximate the area between *a(x)* and the *x*-axis between the end of the school years 1995 and 2002, would you (in general) get a better approximation using 10 left rectangles, 10 right rectangles or 10 midpoint rectangles? **10 midpoint rectangles**

c. What are the width units of the area mentioned in part b? **years**

What are the height units? **hundred applications per year**

What are the area units? **hundred applications**

What does this area represent? (be specific) **The area represents the <u>increase</u> (because *a(x)* is positive from 1995 to 2002) in the number of in-state applications from 1995 to 2002.**

d. Use program NUMINTGL and midpoint rectangles to estimate the accumulated change in in-state applications between the end of the school years 1995 and 2002. Fill in the chart below. Give chart answers to 2 decimal places.

N = number of rectangles	Midpoint area approximation
7	**14.27**
14	**14.45**
28	**14.49**
56	**14.51**
112	**14.51**
256	**14.51**

e. Can you predict, with absolute certainty, what the midpoint area approximation will be for any *N* that is greater than 56? **NO**

- If your answer was *yes*, give the trend.

- If your answer was *no*, choose the QUIT option in program NUMINTGL, restart the program, choose the option to <u>not</u> draw pictures, choose N = 112, record the area in the table above, and continue with larger values of N until you are 100% certain what the midpoint area approximation will be past that point. Once you see a *trend*, put a box around the trend in your table.

So, the trend is __<u>**14.51 hundred applications or 1,451 applications**</u>___ (include units)

f. Interpret your answer to (e) in a sentence.

From the end of the 1995 school year to the end of the 2002 school year the number of in-state applications increased by 1,451 applications.

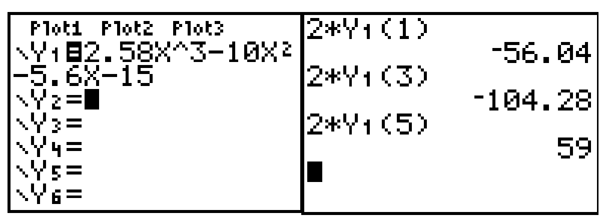
4. The rate of change of the population of a small town, in hundreds of people per year, is modeled by the function P with input t, where t is the number of years since 1993.

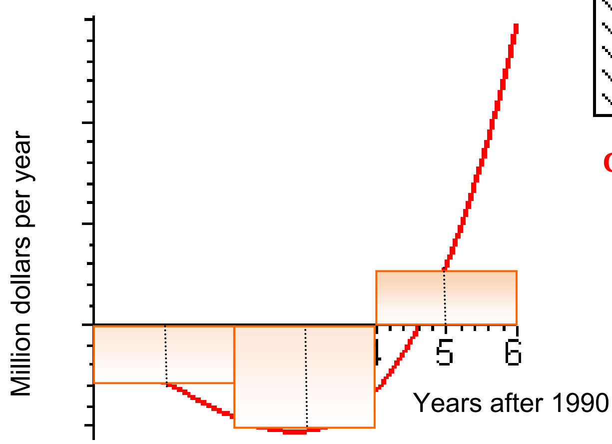
Interpret $\int_2^5 P(t)dt = -14$.

The population of a small town decreased by 14 hundred people from 1995 to 1998.

5. If a company is large enough, its revenue may be thought of as flowing in at a continuous rate. Suppose that it is possible to measure the rate of flow of revenue for a company and that the flow rates measured at the end of the year can be modeled by $r(x) = 2.58x^3 - 10x^2 - 5.6x - 15$ million dollars per year where x is the number of years after 1990.

a. A graph of the function is shown below. Draw 3 midpoint rectangles from x = 0 to x = 6. Use these rectangles to estimate the change in revenue from 1990 to 1996. Include units.





Change in revenue =
-56.04 + -104.28 + 59 = -101.32 million dollars

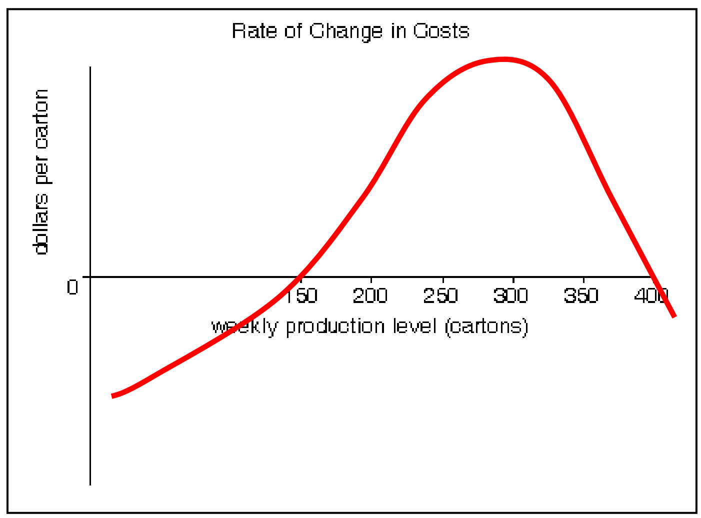
b. Use the idea of a limit of sums to estimate the value of $\int_0^6 r(x)dx$. Round your final answer to two decimal places. Make sure you show a trend and include units with your final answer.

Number of rectangles	$\int_0^6 r(x)dx$
10	-77.260
20	-75.475
40	-75.029
80	-74.917
160	**-74.88**9
320	**-74.88**2
640	**-74.88**1

$\int_0^6 r(x)dx \approx$ **-74.88 million dollars**

c. Interpret your answer to part b.
From 1990 to 1996, the company's revenue decreased $74.88 million.

6. The graph below shows the rate of change of cost for an orchard in Florida at various production levels during grapefruit season. Fill in the blanks in the following cost function discussion. If it is not possible to determine a value, write NA in the corresponding blank.



Cost is increasing when between ___**150**______and ___**400**_______cartons of grapefruit are harvested each week. The cost to produce 200 cartons of grapefruit each week is ___**NA**_______dollars. The cost is lower than nearby costs at a production level of ___**150**_____ cartons, and it is higher than nearby costs at a production level of __**400**____cartons of grapefruit each week. The cost is increasing most rapidly when (about) ___**290**_____cartons are produced each week. The area between the rate-of-change-of-cost function and the production-level axis between the production levels of 250 and 300 cartons each week has units ____**dollars**_________. If $c'(p)$ represents the rate of change of cost (in dollars per carton) at weekly production level of p cartons of grapefruit, would $\int_{330}^{400} c'(p)dp$ be

greater than, less that, or the same value as $\int_{50}^{150} c'(p)dp$? (circle your answer)