Mth Sc 108: Exam 1 Spring 2003 Version 2

Sections 7.2*-7.4*, 7.5, 7.7, 10.1, 4, 6.2, 6.3

For problems 1-3, find the derivative of the function. (6 points each)

1. $g(x) = \log_5(x^3+2) = \dfrac{\ln(x^3+2)}{\ln(5)}$

$$\therefore\ g'(x) = \frac{1}{\ln(5)(x^3+2)} \cdot 3x^2 = \frac{3x^2}{\ln(7)(x^3+2)}$$

2. $y = \ln\left(\sqrt[7]{\dfrac{x^2(x+1)}{(x-3)^2}}\right) = \dfrac{1}{7}\ln\left(\dfrac{x^2(x+1)}{(x-3)^2}\right) = \dfrac{1}{7}\ln(x^2) + \dfrac{1}{7}\ln(x+1) - \dfrac{1}{7}\ln((x-3)^2)$

$$= \frac{2}{7}\ln(x) + \frac{1}{7}\ln(x+1) - \frac{2}{7}\ln(x-3)$$

$$\therefore\ y' = \frac{2}{7x} + \frac{1}{7(x+1)} - \frac{2}{7(x-3)}.$$

3. $y = (2x)^{(1-3x)}$

$$= \exp[\ln((2x)^{1-3x})]$$

$$= \exp\{(1-3x)\ln(2x)]\ .$$

$$\therefore\ y' = \exp[(1-3x)\ln(2x)] \cdot [(1-3x)\tfrac{1}{2x}\cdot 2 + (-3)\ln(2x)]$$

$$= (2x)^{(1-3x)}\left[\frac{1-3x}{x} - 3\ln(2x)\right].$$

-or-

$$\ln(y) = \ln((2x)^{(1-3x)})$$

$$= (1-3x)\ln(2x)$$

$$\therefore\ \frac{y'}{y} = \frac{1-3x}{x} - 3\ln(2x)$$

$$\Rightarrow y' = (3x)^{(1-2x)}\left[\frac{1-3x}{x} - 3\ln(2x)\right]$$

For problems 4-7, evaluate the integral. If the method of substitution is required, be sure to **write out u and du**. For a definite integral give the **exact value**. (8 points each)

4. $\int_0^{\pi/2} \tan\frac{x}{2}\,dx$

Same as Version 1

5. $\int \frac{5x+3}{\sqrt{1-x^2}}\,dx$

$$= \underbrace{\int \frac{5x}{\sqrt{1-x^2}}\,dx}_{\substack{\text{Let } u = 1-x^2 \\ du = -2x\,dx}} + \int \frac{3}{\sqrt{1-x^2}}\,dx$$

$$= -\frac{5}{2}\int u^{-1/2}\,du + 3\arcsin(x) + C$$

$$= -5u^{1/2} + 3\arcsin(x) + C$$

$$= -5(1-x^2)^{1/2} + 3\arcsin(x) + C$$

6. $\int (1+e^{\tan\theta})\sec^2\theta\, d\theta$

Same as Version 1.

Let $u = 3x$
$du = 3dx$

7. $\int \frac{2}{1+9x^2}dx = \int \frac{2}{1+(3x)^2}dx$

$= \frac{2}{3}\int \frac{1}{1+u^2}du = \frac{2}{3}\arctan(u) + C$

$= 2/3 \arctan(3x) + C.$

For problems 8-10, evaluate the limit. If the limit is indeterminate, give its form. Show all work. (6 points each)

8. $\lim_{x\to\infty} \frac{e^{2x}}{3x^2+1} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2e^{2x}}{6x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{4e^{2x}}{6} = \infty.$

9. $\lim_{x\to\pi/2} (\pi/2 - x)\tan x$

Same as Versn 1.

10. $\lim_{x\to 0}(1+2x)^{3/x}$ i.d. form of type 1^∞.

$= \lim_{x\to 0} \exp[\ln[(1+2x)^{3/x}]]$

$= \lim_{x\to 0} \exp[\frac{3}{x}\ln(1+2x)] = \exp[\lim_{x\to 0} \overset{\infty}{\frac{3}{x}} \overset{0}{\ln(1+2x)}]$

$= \exp[\lim_{x\to 0} \frac{3\ln(1+2x)}{x}]$

$\overset{0/0}{=} \exp[\lim_{x\to 0} \frac{\frac{3}{1+2x}(2)}{1}] = \exp[\lim_{x\to 0} \frac{6}{1+2x}]$

$= \exp(6) = e^6.$

For the applications, 11-12, use **exact values**.

11. Find $f(t)$ if $f'(t) = \frac{e^t}{2+e^t}$ and $f(0) = \ln 6$. Show your work. (8 points)

Same as version 1

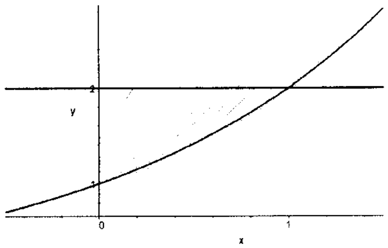
12. At sea level (h=0 km) atmospheric pressure is, on average, 1 atm. On Pike's Peak (h=4.3 km) the atmospheric pressure is about 6/10 atm. Assuming an exponential model for atmospheric pressure, what is the atmospheric pressure at the top of a mountain 3km above sea level? Give the answer in a complete sentence as an exact value. (8 pts)

Same as version 1

NOTE: When asked to set up an integral, do not simplify or evaluate the integral. All limits of integration must be written as exact values.

For problems 13-17, let R be the region bounded by $y = 2^x$, $y = 2$, and $x = 0$.

13. Shade the region on the graph below.



For problems 14-17, set up the integral that gives the volume when the region is revolved around the

14. y-axis using the shell method. (5 points)

Same

15. y-axis using the disk/washer method. (5 points)

Same

16. x-axis. State the method that you use. (5 points)

Same

17. line $y = 2$. State the method that you use. (5 points)

Same