4/9/03, MthSc 108

Quiz 6

Solutions Key

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Name

Answer the following questions. Show analytically if a series is convergent or divergent. If possible, find the series' sum. Show all work. Make sure to show all the hypotheses of any test you use are satisfied. Summarize your conclusion in a sentence form.

Q. Determine convergence or divergence of the following series. Find sum (if possible)

1. $4 - \frac{8}{3} + \frac{16}{9} - \frac{32}{27} + \dots$

Solution

This is a geometric series with $a = 4$, $r = -\frac{2}{3}$
Since $\mid r \mid < 1$ the series is **convergent** by the Geometric Series Test.
Its sum is $s = \frac{a}{1-r} = \frac{4}{1-\frac{-2}{3}} = \frac{12}{5}$

2. $\sum_{n=1}^{\infty} \frac{n}{2n-7}$

Solution

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2n-7} = \frac{1}{2} \neq 0.$$

By the Divergence Test the series is **divergent**.

3. $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$

Solution

Let $b_n = \frac{1}{1+\sqrt{n}}$ and $a_n = \frac{1}{2\sqrt{n}}$. We have

$$a_n = \frac{1}{2\sqrt{n}} = \frac{1}{\sqrt{n}+\sqrt{n}} \leq \frac{1}{1+\sqrt{n}} = b_n$$

Since $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is a divergent p-series ($p = \frac{1}{2}$)
Then, by the Direct Comparison Test, the series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ is also **divergent**

4. $\sum_{n=1}^{\infty} ne^{-n}$

Solution

This series reminds one of the integrals we saw earlier in the course. We can try the Integral Test. Let $f(x) = xe^{-x}$. The function is

1. continuous (as a product of two continuous functions)
2. positive, for all $x > 0$
3. decreasing, for all $x > 1$ (since $f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x) < 0$ for all $x > 1$)

Also $a_n = f(n)$ and $\int_1^{\infty} xe^{-x}dx = \frac{2}{e}$ (i.e. convergent) therefore by the Integral Test the series $\sum_{n=1}^{\infty} ne^{-n}$ is also **convergent**.