Use the method of separation of variables to show that the 1-D heat conduction problem: find $u = u(x, y)$ so that

$$
\frac{\partial u}{\partial t}(x,t) - \alpha \frac{\partial^2 u}{\partial x^2}(x,t) = 0, \quad 0 < x < l, \quad t > 0
$$

$$
u(0,t) = 0, \quad t \geq 0
$$

$$
u(l,t) = 0, \quad t \geq 0
$$

$$
u(x,0) = f(x), \quad 0 \leq x \leq l
$$

has the solution

$$
u(x,t) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \exp \left[-\alpha \left(\frac{n\pi}{l}\right)^2 t\right]
$$

where

$$
c_n = \frac{2}{l} \int_{0}^{l} f(x) \sin \frac{n\pi x}{l} \, dx.
$$