Use the method of separation of variables to show that the 1-D heat conduction problem: find $u = u(x, y)$ so that

$$\frac{\partial u}{\partial t} (x,t) - \alpha \frac{\partial^2 u}{\partial x^2} (x,t) = 0, \quad 0 < x < l, \ t > 0$$

$u(0,t) = 0, \ t \geq 0$

$u(l,t) = 0, \ t \geq 0$

$u(x,0) = f(x), \ 0 \leq x \leq l$

has the solution

$$u(x, t) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \exp \left[ -\alpha \left( \frac{n\pi}{l} \right)^2 t \right]$$

where

$$c_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} \ dx.$$