Use the method of separation of variables to show that the 1-D heat conduction problem:
find $u = u(x, y)$ so that

\[ \frac{\partial u}{\partial t}(x, t) - \alpha \frac{\partial^2 u}{\partial x^2}(x, t) = 0, \; 0 < x < l, \; t > 0 \]

\[ u(0, t) = 0, \; t \geq 0 \]

\[ u(l, t) = 0, \; t \geq 0 \]

\[ u(x, 0) = f(x), \; 0 \leq x \leq l \]

has the solution

\[ u(x, t) = \sum_{n=1}^{\infty} c_n \sin \left( \frac{n\pi x}{l} \right) \exp \left[ -\alpha \left( \frac{n\pi}{l} \right)^2 t \right] \]

where

\[ c_n = \frac{2}{l} \int_0^l f(x) \sin \left( \frac{n\pi x}{l} \right) dx. \]