The Research Base for Math Out of the Box™

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Math Out of the Box™ is a K-5 mathematics curriculum which is under development in the College of Engineering and Science at Clemson University and is published by Carolina Biological Supply Company, Inc. Math Out of the Box™ is research and standards based. The name derives from the packaging of Math Out of the Box™ in "kits" that contain all the materials teachers will need to teach this inquiry-based curriculum and from the idea that those using this innovative curriculum will be "thinking outside the box" of traditional instructional approaches in mathematics. The curriculum is designed to be released in four vertical strands in the following order: *Developing Algebraic Thinking, Developing Geometric Logic, Developing Measurement Benchmarks, and Developing Number Concepts*.

Math Out of the Box™ is developed through a rigorous process of research, development, lesson testing, and revision. The body of research on teaching and learning reviewed in the initial phase of the project included publications such as the following: *How People Learn: Brain, Mind, Experience, and School* edited by John D. Bransford, Ann L. Brown, and Rodney R. Cocking: *Eager to Learn: Educating Our Preschoolers* edited by Barbara T. Bowman, M. Suzanne Donovan, and M. Susan Burns; and *Understanding by Design* by Grant Wiggins and Jay McTighe. Other sources which focused on the teaching and learning of mathematics included in the body of preliminary research were *Adding It Up: Helping Children Learn Mathematics* edited by Jeremy Kilpatrick, Jane Swafford, and Bradford Findell and the *Handbook of International Research in Mathematics Education* edited by Lyn D. English.

Standards and goals documents were reviewed as part of the research phase. The documents included the following:

- Project 2061, American Association for the Advancement of Science. *Benchmarks for Science Literacy.* New York: Oxford University Press. 1993.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* 2000.
- National Research Council. *Inquiry and the National Science Education Standards: A Guide for Teaching and Learning.* 2000.
- National Research Council. *National Science Education Standards.* 1996.
- International Society for Technology in Education. *National Educational Technology Standards.* 2000.

As the development phase of the curriculum continues, mathematical topics are chosen and sequenced vertically between the grades and horizontally within each grade level resulting in a connected curriculum. Researchers who participated in the Third International Math and Science Study (TIMSS) conclude that "The particular topics that are presented at each grade level, the sequence in which those topics are presented, and

the depth into which the teacher goes are all critical decisions surrounding the curriculum that have major implications for what children learn" (Schmidt, Houang, and Cogan, 2002 p 15).

The Math Out of the Box™ curriculum is researched and developed by teachers, under the guidance of specialists in science and mathematics reform. It is, by designed, an inquiry-based curriculum accessible to students, teachers, and families. During the development phase, teams of teachers and other practitioners, representing all levels of mathematics teaching, from pre-school to post-secondary education, design and review lessons. The teachers involved in development of the lessons bring experiences in mathematics teaching and learning that range from traditional mathematics practices to innovative reform curricula implementation.

As each grade level of a Math Out of the Box™ strand is developed, a grade level field test is implemented. The field test begins with professional development for field test teachers, a school-based coach, and their school district mathematics coordinator. As lessons and materials are field tested, information is gathered through assessment items, teacher reflections, videos, student work samples, parent feedback, pre/post tests, and anecdotal records. The publisher's project team continues to monitor lessons and materials through the field-test phase.

After the field test in diverse classrooms, lessons are again reviewed and revised by teachers and specialists. Selected lessons are retested in classrooms before another round of revision under the guidance of the editorial and layout team of the publisher, Carolina Biological Supply Company. This development process which includes researchers, teachers, specialists, and diverse classroom settings results in a unique curriculum built around sub-concepts that connect to big mathematical ideas.

The developers of Math Out of the Box™ have extensive experiences in teaching at all levels of K-16 education. The developers, as active participants in the decade-long South Carolina Statewide Systemic Initiative, have extensive experience in designing and delivering professional development for education leaders, community organizations, and families. These professional development experiences include in-service and pre-service classes and workshops for mathematics and science teachers. As a result of these experiences in mathematics and science reform, the developers of the Math Out of the Box™ curriculum formed the following beliefs about mathematics teaching and learning which are supported by the research-base of the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000):

- All students must have access to a curriculum that connects mathematical ideas.
- All teachers of mathematics need to be confident in their own teaching and learning as well as that of their students.
- Students need to have rich and varied experiences and materials as part of their mathematical learning.
- Assessment guides students in knowing what they have learned, aids teachers in planning instruction, and informs the community.

- Technology supports students and teachers as they engage in rich mathematical experiences.

Reports from both public and private research groups such as the *Status of Elementary School Mathematics Teaching* by Horizon Research, Inc. (Malzahn, 2002); *A Study of K-12 Mathematics and Science Education in the United States* by Horizon Research, Inc. (Weiss, 2003); *What It Takes: Pre-K-12 Design Principles to Broaden Participation in Science, Technology, Engineering and Mathematics* by Building Engineering and Science Talent (BEST, 2004); and *Highlights From the Trends in International Mathematics and Science Study (TIMSS) 2003* (Gonzales, 2004) provide information about the state of mathematics education in the United States and other countries.

Achievement gaps such as those cited by the Southern Regional Education Board in its 2004 report existed in the 1990s and continue to exist with little evidence of significant progress. In the eleven states included in the Southern Regional Education Board report, "Neither the percentages of black nor Hispanic students who met state standards in any state equaled the percentages of white students who met state standards. The percentages of black students who met state standards in mathematics trailed the percentages of white students who met state standards by between five and 36 percentage points in those 11 states" (Lord, Wade, and Creech, 2004, 10). The developers of the *Principles and Standards* stated, "All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence" (NCTM, 2000, p.50).

The ability to reason mathematically, is needed by consumers in making informed choices, by citizens in making good decisions, and by all students in order to succeed in reaching education and career goals. Mathematics helps people appreciate patterns and symmetry in the world around them and to better understand the beauty found in different cultures (Kilpatrick, Swafford, and Findell, 2001; Van de Walle, 2004; Devlin, 2000).

To increase the successful teaching and learning of mathematics for all teachers and students, the developers of the Math Out of the Box™ curriculum included specific components considered to be essential to the implementation of the curriculum and to closing achievement gaps. Following are the descriptions and the research-base for those components essential to fulfilling the mathematical promise in every child.

A Community of Learners

Math Out of the Box™ lessons are designed to build a community of learners in the classroom where both teachers and students respectfully share ideas and question each other. One way of building community in a mathematics classroom is with the use of a variety of group configurations. Math Out of the Box™ lessons provide a structure for teachers in organizing different collaborative groups throughout each lesson.

At the beginning of a lesson, students and the teacher work in a whole group to pose questions, define problems, brainstorm ideas, and discuss solutions to lay the organizational groundwork that leads to further investigation. Later in the lesson, students

share, in small groups, experiences that challenge them to solve problems. Next, each student becomes accountable for his or her own learning as students again meet in a whole group to share and reflect on each other's ideas. This explicit representation and sharing of ideas with others increases the likelihood that students will connect what they have learned with what they already know and retain their learning (Pellegrino, Chudowsky, and Glaser, 2001).

Extensive research corroborates the effectiveness of collaborative groups in K-5 classrooms. After examining the large body of research on cooperative groups, one group of researchers conclude that "Markedly different theoretical perspectives (social interdependence, cognitive-developmental, and behavioral learning) provide a clear rationale as to why cooperative efforts are essential for maximizing learning and ensuring healthy cognitive and social development as well as many other instructional outcomes" (Johnson, Johnson, and Stanne, 2000, p.9).

Another learning community addressed in Math Out of the Box™ lessons is the community of teachers as learners. Ideally, teachers plan and reflect together as a support for their own teaching and learning. This collegial support is an important factor in successful implementation of an innovation (Elly, 1990, Diaz, 2004). In reality, the relationships within a group of teachers at a grade level and within a school system are complex (Sarama, 1998). To promote collegial discussion and reflection, Math Out of the Box™ lessons include reflective questions that serve as discussion starters for teachers as learners as they implement the curriculum.

Making Mathematics Meaningful With Connections

The ability to recognize relationships among mathematical ideas and to apply those ideas beyond the mathematics classroom has long been recognized as a hallmark of mathematical understanding (Brownell, 1954; Skemp, 1978; Grouws & Cebulla; 2000). In recent years, the ability to recognize such relationships is often referred to as "making mathematical connections." The benefits of mathematical connections in developing mathematical understanding is well documented in cognitive psychology, and is recognized as an essential part of learning mathematics by mathematics teachers and educators (Stigler and Hiebert, 2004). In the 2004 TIMSS Video Study, the making of connections among mathematical ideas was cited as the most significant feature distinguishing the higher-achieving countries' mathematics instruction from the other countries in the study (Stigler and Hiebert, 2004). The importance of making mathematical connections in developing mathematical fluency cannot be overstated.

The Math Out of the Box™ curriculum is designed so that students will develop the ability to make the following mathematical connections:

- among mathematical ideas, facts, and procedures.
- between mathematical ideas and other disciplines.
- to their own environment.

This is accomplished through an intentional horizontal and vertical design that supports students in discovering and applying important mathematical relationships. Connections are developed horizontally through a "storyline" of related mathematical sub-concepts within a grade level. The horizontal organization of the age-appropriate tasks provides opportunities for students to make coherent and meaningful connections among mathematical ideas. Connections are developed vertically through the grade levels using a K-5 content strand curriculum design. The vertical design of the curriculum through the content strands provides opportunities for students to develop broader and deeper understandings of the "big" mathematical ideas as they progress from one grade to the next. This horizontal and vertical weaving of the mathematical ideas supports students in building a mathematical system that represents a coherent and comprehensive whole rather than a series of isolated skills, facts, and rules. In the Math Out of the Box™ curriculum, skills, facts, and rules are learned and practiced so that students develop a relational understanding among the skills, facts, and rules, and the mathematical ideas that connect them.

In addition to making mathematical connections among mathematical ideas, the Math Out of the Box™ curriculum supports students and teachers in making connections between mathematics and other disciplines and between mathematics and their environment. The writing model that is embedded throughout every lesson provides opportunities for students to develop both oral and written communication skills in an environment that fosters creative thought. The Apply phase of the Learning Cycle found in each lesson provides a variety of alternative suggestions for teachers to use in connecting the content of the lesson to literature, social studies, science, art, music, and physical education. Home Connections are included throughout the units to provide opportunities for families and others in the community to become engaged in the mathematical learning of the students.

Making mathematical connections is an inherent part of the design of the Math Out of the Box™ curriculum. Whether among mathematical ideas, to other disciplines, or to the real world, the connections that students make in the Math Out of the Box™ lessons support the students in developing the mathematical promise that exists in each of them.

Communicating About Mathematics

Discussion, questioning, reflection, and writing are communication strategies that ensure that meaningful mathematical thinking occurs in mathematics classrooms. Communication in the mathematics classroom permits learning to build on the students' informal knowledge, gives students practice in explaining their mathematical thinking to others, and provides students and teachers with evidence that learning has occurred. (Yackel , Cobb, Wood, and Merkel, 1990; Malloy, 1997; Moody, 2004).

The communication model in Math Out of the Box™ lessons provides a structure for successful verbal and written experiences throughout each mathematical subconcept. This model results in a community in which students have the freedom to take risks so that verbal and written communication can occur and develop. In Math Out of the Box™

lessons, communication evolves and improves as discussion and writing moves from part of a community to individual accountability.

Lessons begin with whole-group communication where students and teachers brainstorm together to develop a representation or generalization of a mathematical concept. In this Engage phase of the Learning Cycle, the teacher can assess the prior learning of the student. Later in the Investigate phase, students work in collaborative groups permitting the mathematical thinking to evolve in the safe environment of collaboration. After each student writes to show his or her own mathematical thinking, another whole group discussion takes place in which ideas are shared, changes in thinking are discussed, and ownership of the learning takes place.

The Math Out of the Box™ communication model is based on research that considers the reflection process to be essential to the process of learning (Schon, 1983; Confrey, 1990; Moon, 1999). Schon described two types of reflection—reflection-in-action and reflection-on-action (1983). The first is sometimes described as "thinking on our feet," and the second requires exploring why we thought the way we did. Math Out of the Box™ lessons are designed to promote both types of reflection. First, the Learning Cycle provides a structure for continuous reflection-in-action as students represent, communicate, and compare their findings throughout each lesson. Second, the Reflect phase of the Learning Cycle provides an opportunity for focused reflection-on-action as students are asked to examine and explain their thinking. It is through this linked process of reflection, in-action and on-action, that students take ownership of their learning.

The communication model is also based on research that recognizes the impact of language on mathematical understanding and concept development. The uniqueness of mathematical language requires special attention to its development in children. Many sources indicate that success in mathematics is best accomplished through activities that use the language of mathematics as a tool for building mathematical thinking (Baker, 1990; The Mathematical Association, 1992; Brickmore-Brand, 1993; Usiskin, 1996; Whitin, 2000). Math Out of the Box™ lessons encourage the development of correct mathematical vocabulary by providing opportunities for students to explore a mathematical concept through a variety of concrete experiences and by routinely encouraging students to communicate their understanding using mathematical language that is developmentally appropriate for the student.

In Math Out of the Box™, questions for different purposes and at different levels of thinking are provided so that students will connect present learning with past learning, share their ideas with each other, develop the ability to think critically, and be responsible for their own learning (Anderson et al., 2001). Research shows that questions are an effective learning tool when teachers use effective questioning techniques. Questions help students cement skills, explore concepts, and self-assess their own knowledge. A safe environment for asking and answering questions is essential to success in the mathematics classroom. Confident, secure teachers are comfortable with the fact that discussion is not going to be predictable and that they may not always know the answer (Bransford, Brown, and Cocking, 2000; Marzano, Pickering, and Pollack, 2001).

Balanced Assessment That Informs the Mathematical Community

Assessment is an ongoing, essential component of the inquiry-based learning cycle used in the Math Out of the Box™ curriculum. Assessments are built around concepts and skills based on mathematical standards. The goals for assessment in the Math Out of the Box™ curriculum are

- to guide students in knowing what they have learned.
- to allow the teacher to understand how students are thinking about mathematics.
- to aid teachers in planning instruction.
- to inform the community.

Two types of assessment are used throughout the lessons. Formative assessments are embedded into the lessons, providing information to the teacher for instructional decisions and information to the students about their own learning. Numerous studies support the practice of formative assessment as a way to increase student success, particularly with low-achieving students (Fuchs and Fuchs, 1986; Wiliam and Black, 1996). Summative assessments provide additional information about student learning and can be evaluative in nature, providing information to a broader community. A variety of assessment strategies are included in each lesson to allow students multiple opportunities to demonstrate their knowledge and skills. Lessons that function as performance assessments are included in each module.

An important aspect of assessment in Math Out of the Box™ lessons is the opportunity for students to be involved in their own assessment in each lesson. Through innovative tools such as the Bright Idea Pencil, students assess their own and each other's learning. Based on these assessments, they add to or make changes in their mathematical writing. Leading researchers cite formative assessment that includes self-assessment by students as one type of classroom assessment that is most likely to enhance student achievement (Black and Wiliam, 1998; Stiggins, 2001).

The development of new mathematical ideas in Math Out of the Box™ lessons is supported by the variety of materials available for exploration and modeling. When students test, modify, and reflect on ideas, an opportunity for formative assessment exists as students use manipulatives as a means to integrate ideas with prior learning (Van de Walle, 2004).

A Variety of Problem Solving Strategies

Data from reform curricula of the 1990s indicate that students using curricula, with an emphasis on problem solving, perform as well as students using traditional curricula on basic skills and better on conceptual understanding on standardized tests (Schoenfeld, 2002; Senk and Thompson, 2003). Research indicates that opportunities to explore new ideas balanced with opportunities to practice skills results in successful problem solving (Grouws and Cebulla, 2000).

Researchers continue to find a relationship between the development of students as thinkers and student success in problem solving and conceptual understanding. Studies have examined the issues in classroom application when problem solving is considered as a process rather than another topic in a mathematics curriculum (Fennema, Carpenter, and Lamon, 1996; Kazemi, 1998; Kennedy, Tipps, and Johnson, 2004). The following beliefs of the developers of the Math Out of the Box™ curriculum are based on such research:

- A safe environment must be developed as part of the learning community so that mathematical discourse can take place.
- Changes in thinking can occur as errors and misconceptions are reconceptualized.
- Successful problem solving often requires multiple attempts and multiple strategies.
- Problem solving as a community leads to shared understanding of mathematical ideas, individual accountability, and connections to life outside of the mathematics classroom.

In Math Out of the Box™ lessons, students are provided with real-life and simulated situations so that making conjectures and developing proofs can lead to correct mathematical reasoning. Problem solving opportunities are enriched with rehearsals or explicit connections to prior learning at the beginning of each unit of a strand, at the beginning of each subconcept, and at the beginning of each lesson. Each lesson is developed to provide whole group, small group, and individual opportunities for discovering, building, and experiencing problem solving. Materials, manipulatives, and models are used as tools to develop visual representations for problem solving. Opportunities to practice skills and problem solving strategies using a variety of these representations are provided in each lesson.

Inquiry-Based Learning Cycle

The idea of organizing inquiry processes around a learning cycle has its research base in inquiry science (Karplus and Thier, 1967; Lawson, Abraham, and Renner, 1989). When learning cycles are used in curriculum materials, “students develop higher-level thinking skills and sound understanding of concepts” (Marek and Cavello, 1997, p. 123).

The developers of the Math Out of the Box™ curriculum chose to use a learning cycle in each lesson to provide teachers with a template that promotes the development of active inquiry and critical thinking. The four phases of the Math Out of the Box™ Learning Cycle are Engage, Investigate, Reflect, and Apply.

The Engage phase of the learning cycle allows students to make connections between past and present learning experiences and also provides a natural pre-assessment for both the teacher and the student. This type of engagement, according to brain research, helps ensure retention of the new learning (Lowery, 1998; Wolfe, 2001). In the Engage phase,

the class is involved in asking questions, defining problems, brainstorming ideas, and discussing solutions. Teachers engage students' natural curiosity and lay the groundwork for further investigation.

The Investigate phase of the learning cycle is based in the "cognitive principle of assimilation" (Baroody & Ginsburg, 1990, p. 56). This principle implies that understanding cannot be imposed on the learner, but instead is developed progressively by the learner, beginning with concrete and progressing to abstract opportunities.

The reflective process, according to Confrey, is "essential" to the process of learning (1990, p. 109). Research, experimentation, observation, building models, and redefining questions are all part of the Investigate phase of the learning cycle. During the Reflect phase of the cycle, the role of the teacher is especially critical because it is the teacher's mathematical knowledge that enables the teacher to assist his or her students in summarizing and structuring their thinking into meaningful models of the mathematical ideas they have investigated (Confrey, 1990). In Math Out of the Box™ lessons, the Reflect phase of the Learning Cycle provides the opportunity for students to share ideas with others and to more formerly connect what they have learned with what they already know.

In the fourth phase of the cycle students are challenged to apply their newly acquired knowledge to slightly different situations or to explore broader or deeper applications of their discoveries. In this Apply phase of the cycle, both students and teachers can assess the depth of understanding of the newly formed ideas as the new knowledge becomes prior knowledge on which to connect new learning--and the cycle of learning begins again (Diaz, 2004).

Materials, Manipulatives, and Models

Researchers advocate an environment of hands-on experiences in mathematics classrooms. In addition to manipulatives, materials needed for this rich environment include charts, graphs, writing models, diagrams, technology, and any tool that aids students in sense-making and problem solving (Sowell, 1989; Hiebert et al., 1997; Kilpatrick, Swafford, and Findell, 2001; Van de Walle, 2004).

Each Math Out of the Box™ unit includes a teacher's manual with student blackline masters and a kit of materials needed to effectively teach the lessons. Including the materials as part of the curriculum and in professional development, ensures that materials are used effectively by students to demonstrate and develop knowledge, to self-assess learning, and to connect mathematic ideas. Embedding the use of materials throughout the learning cycle of each lesson provides a powerful means of formative assessment for the teacher as students investigate mathematical ideas.

Research, experimentation, observation, building models, and redefining questions are all part of investigation. When students are given common, concrete experiences that challenge them to solve problems, information is gathered, patterns observed and

analyzed, connections are made and applied, and conclusions are drawn and defended. The result is mathematical reasoning and increased understanding of mathematics.

Professional Development

Innovative and meaningful professional development experiences provided in partnership with established organizational structures are needed for successful implementation of any curriculum (DuFour and Baker, 1998; Loucks-Horsley, Hewson, Love, and Stiles,1998; Guskey, 2000). As strands of the Math Out of the Box™ curriculum are developed, correlating professional development experiences in partnership with the publisher and other organizations are also developed, field-tested, and revised.

In addition to these professional development experiences, the Math Out of the Box™ curriculum includes embedded strategies to support and change teachers' knowledge and beliefs about mathematics. Throughout the lessons, procedures and processes of effective teaching are modeled for teachers including effective questioning, writing strategies, representation as a key to successful problem-solving, and reflective practices. The learning cycle that is used to organize the lessons in the Math Out of the Box™ curriculum provides teachers with an effective mechanism for including inquiry-based practices, such as formative and summative assessments, throughout the mathematics lessons. Research shows that teachers' knowledge and belief systems can be affected by such experiences (Fullan, 1982; Cohen and Ball, 1990; Fennema, Carpenter, and Lamon, 1991).

Conclusions

The development of the Math Out of the Box™ curriculum has been influenced by the classroom experiences of the developers, by the professional development experiences of specialists in mathematics and science reform, and by a rich body of research about the teaching and learning of mathematics. The components considered by the developers to be essential to the successful implementation of the curriculum include the following:

- the development of a community of learners.
- explicit connections that make mathematics meaningful.
- a model for verbal and written communication.
- balanced assessment practices.
- a variety of problem solving experiences.
- a learning cycle that provides a structure for inquiry throughout each lesson.
- opportunities for investigation of mathematical ideas with a diversity of materials, manipulatives, and models.
- professional development that is designed to improve teachers' mathematical knowledge for teaching.

Continued Development and Implementation

The National Science Foundation funded curricula of the 1990s increased the body of research on the teaching and learning of mathematics. The developers of the Math Out of the Box™ curriculum hope to add to that rich body of research in the following ways:

- Continue to inform the curriculum by a rigorous cycle of researching, developing, testing, and collecting and analyzing data through field test and pilot projects. The developers of the Math Out of the Box™ curriculum wish to thank all of the school district leaders and classroom teachers who have supported the research and development of this mathematics reform project. As of February 2005, Math Out of the Box™ lessons have been tested in 3 states, in 11 school districts, and in over 100 classrooms.

- Continue to develop partnerships with corporate partners, foundations, and others who provide funding, networking opportunities, evaluation, and advice on addressing the problems of mathematics education in the United States. The developers of Math Out of the Box™ thank the following organizations for their support of this innovative curriculum development project: the College of Engineering and Science at Clemson University including the Department of Mathematical Sciences and the Center for Excellence in Mathematics and Science Education, Carolina Biological Supply, Inc., South Carolina's Coalition for Mathematics and Science (SCCMS), DuPont, American Honda Foundation, Michelin North America, the General Electric Foundation, SECME at Georgia Institute of Technology, and the Call Me Mister® project in the Eugene T. Moore School of Education at Clemson University.

- Continue to collect and analyze data to inform the research and development of the curriculum and to inform the greater educational community. Preliminary data collected during field tests of the Developing Algebraic Thinking Strand of the curriculum indicate that Math Out of the Box™ has the potential to be instrumental in closing achievement gaps. Pilot projects with outside evaluators instituting scientific research studies are being planned for various geographic regions of the United States. Currently under development are a statewide initiative in Delaware, a pilot with a consortium of school districts in western Pennsylvania, and a research study in a school district affiliated with Princeton University. These studies and others (e.g., dissertations, journal publications, and Math Out of the Box™ technical reports) will add to the growing body of knowledge about Math Out of the Box™ and the teaching and learning of mathematics.

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