

Computational and Experimental Mathematics: Intermediate Explorations

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Doing mathematics requires active involvement

It is often remarked that “mathematics is not a spectator sport”. Indeed, the first occurrence of “the proof is left as an exercise” occurred in *De Triangulis Omnimodis* by Regiomontanus, written 1464 and published 1533. He is quoted as saying “This is seen to be the converse of the preceding. Moreover, it has a straightforward proof, as did the preceding. Whereupon I leave it to you for homework.”

Gauss, Ramanujan, and many other great mathematicians were prodigious calculators: programs such as Sage, Maple, Mathematica, etc. enable us to begin to do the same.

This session will have two components

- A hands on introduction to SAGE,
a new and impressive computational tool for math
- An exploration of some experimental mathematics

This intermediate session will be accessible to those with some knowledge of undergraduate mathematics: the topics may include some unfamiliar ideas, but all the techniques will be elementary.

Last session we

- Introduced Sage as a tool for exploring mathematics
- Set up accounts at Clemson's sage server
- Learned how to open worksheets and edit a copy
- Explored some properties of Pascal's triangle
- Introduced some of Sage's calculus tools

To come in this session

This session we will

- Learn more about the Sage interface
- Continue exploring calculus and differential equations
- Learn how to visualize data in Sage
- Explore continued fractions of rationals and irrationals
- Explore Padé approximants for functions

Throughout the session, participants are encouraged to work through the examples, and to experiment with “what if” questions.

Setting up a Sage session

To get started with Sage open a browser, preferably Firefox, and

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`https://clemix.clemson.edu:34567/`

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- Log in to your account (create one if you have not already done so)
- Click on Published
- Select the worksheet SC10CE2a
- Click on “Edit a copy”

The Sage Intermediate 1 Worksheet

Recall

- The Sage notebook uses blocks of text essentially Python statements that get executed
- Lines that start with the pound sign are comments
- A block is evaluated by clicking on the evaluate link or by typing shift-enter

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- Sage can handle rational functions
- Sage will often provide answers that are correct, but require an understanding of the context (for example: logarithms as functions of the complex plane)
- Sage's graphics capabilities can provide helpful illustrations

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- ODEs, symbolic solution

Differential Equations

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- ODEs, plot several solutions on the slope field
- Systems of ODEs, numerical solution
- Systems of ODEs, plot several solutions on the same vector field

Making good use of calculus

A slice of π .

- $\pi \neq 22/7$.

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- $\pi < 22/7$: a rigorous proof!
- A series for π
- An extremely accurate, inaccurate approximation!

Some sample explorations using Sage

- $\exp(x) = 0$ has no roots: what about its taylor polynomials?
- The Euclidean Algorithm on real numbers
- Padé Approximation for functions

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And now:

The book giveaway.

Once again, our thanks to

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- The SuperComputing 2010 Conference, the Education Program organizers, and all the affiliated sponsors.
- You, the audience, for your attention