Section 1.1 Review of Functions

Domain and Range

A function f assigns a unique output f (x) for each input x.

The set of all possible input values is called the domain and the set of all possible output values (given the domain) is called the range.

The independent variable is the variable associated with the domain.

The dependent variable belongs to the range.

EX: In the function y = f (x),

x is the   variable and

y is the   variable.

EX: p. 8, #16 State the domain and range of the function.

$$f(w) = \sqrt[4]{2-w}$$

Domain and Range in Context

If the domain is not specified, we take it to be the set of all input values for which the function is defined.

NOTE: The domain and range of a function may be restricted by the context of the problem. See Example 3, p. 3.

EX: p. 8, #20 Determine an appropriate domain of each function. Identify the independent and dependent variables.

The average production cost for a company to make n bicycles is given by the function $c(n) = 120 - 0.25n$.

Domain (in context):

Independent variable:

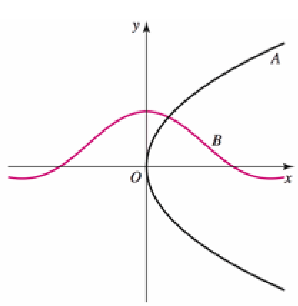
Dependent variable:

Graphs of Functions

The graph of a function f is the set of all points (x, y) in the xy-plane that satisfy the equation $y = f(x)$.

A graph represents a function if and only if it passes the Vertical Line Test: Every vertical line intersects the graph at most once.

EX: p. 7, #11 Decide whether graph A, graph B, or both graphs represent functions.



Graph A: function not a function

Graph B: function not a function

Composite Functions

Given two functions f and g, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$.

Domain: Domain of g for which g (x) is in the domain of f.

The function $y = f(g(x))$ should be evaluated in two steps.

1st: Evaluate $u = g(x)$.
2nd: Evaluate $y = f(u)$.

EX: p. 8, #25 Simplify or evaluate the following expressions.

$f(x) = x^2 - 4$, $g(x) = x^3$, $F(x) = 1/(x - 3)$

25. $F(g(y))$

Extra. $F(f(g(2)))$

EX: p. 8, #32 Find possible choices for outer and inner functions f and g such that the given function h equals $f \circ g$. Give the domain of h.

$$h(x) = \frac{2}{\left(x^6 + x^2 + 1\right)^2}$$

Symmetry in Graphs

A graph is symmetric about the y-axis if whenever the point (x, y) is on the graph, the point (-x, y) is also on the graph.

An even function f has the property that f (-x) = f (x) for all x in the domain. The graph of an even function is symmetric about the y-axis.

A graph is symmetric about the origin if whenever the point (x, y) is on the graph, the point (-x, -y) is also on the graph.

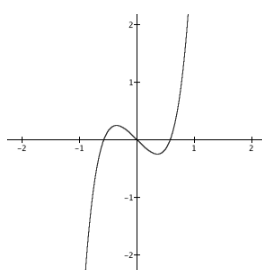
An odd function f has the property that f (-x) = - f (x) for all x in the domain. The graph of an odd function is symmetric about the origin.

A graph is symmetric about the x-axis if whenever the point (x, y) is on the graph, the point (x, -y) is also on the graph.

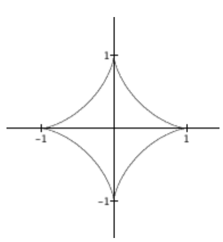
NOTE: A graph which is symmetric about the x-axis cannot represent a function.

EX: p. 8, #48, 51 Determine whether the graphs of the following equations and functions have symmetry about the x-axis, the y-axis, or the origin.

#48 $f(x) = 3x^5 + 2x^3 - x$



#51 $x^{2/3} + y^{2/3} = 1$



Section 1.2 Representing Functions

Using Formulas, p. 10 (Function Types)

- **Polynomials**: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

  $a_n, a_{n-1}, \ldots, a_1, a_0$ are real coefficients with $a_n \neq 0$.

  n is a nonnegative integer called the degree.
  All polynomials have domain $(-\infty, \infty)$.

- **Linear** Functions: $f(x) = mx + b$ $y = mx + b$ is slope-intercept form.

  m is the slope (rise/run)
  b is the y-intercept ⟶ $(0, b)$ is a point on the graph.

  $m > 0$ ⟶ line goes up from left to right (increasing function)
  $m < 0$ ⟶ line goes down from left to right (decreasing fnc)
  $m = 0$ ⟶ line is horizontal

Function Types (Continued)

- **Power** Functions: $f(x) = x^n$ where n is a positive integer.

  See p. 13, Figures 1.21 and 1.22.

- **Root** Functions: $f(x) = x^{1/n}$ where $n > 1$ is a positive integer.

  See p. 13, Figures 1.23 and 1.24.

- **Rational** Functions: Functions which are a quotient or ratio of polynomials: $f(x) = p(x) / q(x)$, where p and q are polynomials.

- **Algebraic** Functions: Functions constructed from polynomials using algebraic operations.
  (addition, subtraction, multiplication, division, taking roots)

Function Types (Continued)

- **Exponential Functions:** $f(x) = b^x$ where b is a positive constant and $b \neq 1$. b is called the base.

  All exponential functions have domain $(-\infty, \infty)$
  Most important is the natural exponential: $f(x) = e^x$.

- **Logarithmic Functions:** $f(x) = \log_b x$ where $b > 0$ and $b \neq 1$.

  All logarithmic functions have domain $(0, \infty)$
  Most important is the natural logarithm: $f(x) = \log_e x = \ln x$.

- **Trigonometric Functions:** For example, $f(x) = \sin x$ or $f(x) = \cos x$.
  These will be reviewed in Section 1.3.

- **Transcendental Functions:** Functions that are not algebraic.

Piecewise Functions

Functions that have different definitions on different parts of the domain are called piecewise functions.

If all of the pieces are linear, the function is piecewise linear.

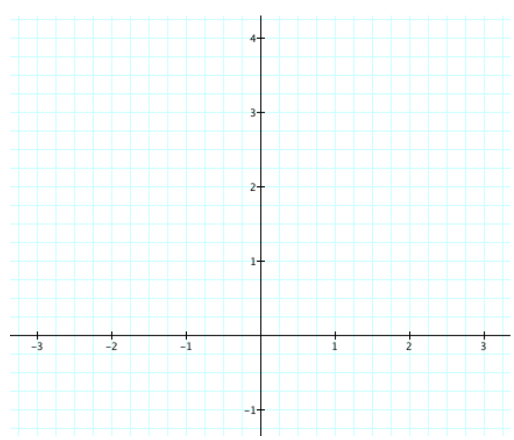
EX: p. 12, Example 4b

$f(x) = |x|$

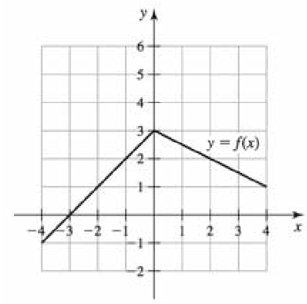
$|-3| =$ $\qquad$ $|3| =$

$f(x) = \begin{cases} \quad , & \text{if } x < 0 \\ \quad , & \text{if } x \geq 0 \end{cases}$

f (x) is increasing on
and decreasing on



EX: p. 19, #15 **Write a definition of the function whose graph is given.**



f (x) = {

p. 19, #25 **Determine the slope function for the function found in Exercise #15. (See p. 15, Example 6.)**

g (x) = {

Transformations of Functions and Graphs

SUMMARY **Transformations**

Given the real numbers a, b, c, and d and the function f, the graph of $y = c\,f(a\,(x - b)) + d$ is obtained from the graph of $y = f(x)$ in the following steps.

$$y = f(x) \xrightarrow{\text{horizontal scaling by a factor of } |a|} y = f(ax)$$
$$\xrightarrow{\text{horizontal shift by } b \text{ units}} y = f(a(x - b))$$
$$\xrightarrow{\text{vertical scaling by a factor of } |c|} y = c\,f(a(x - b))$$
$$\xrightarrow{\text{vertical shift by } d \text{ units}} y = c\,f(a(x - b)) + d$$

NOTE: **See p. 17, Figures 1.34 - 1.39. (MyMathLab Interactive eBook)**

Horizontal scaling by a factor of | a |: y = f (ax)

0 < | a | < 1 → f is <u>broadened</u> horizontally
| a | > 1 → f is <u>steepened</u> horizontally

Vertical scaling by a factor of | c |: y = c f (x)

0 < | c | < 1 → f is <u>broadened</u> vertically
| c | > 1 → f is <u>steepened</u> vertically

EX: Give an equation for the graph.

$y = x^3$ shifted left 1, down 1
and reflected about the y-axis

EX: Give an equation for the graph.

$y = 1 + \frac{1}{x^2}$ scaled horizontally by a factor of 1/2 (broaden)

$y = 1 + \frac{1}{x^2}$ scaled vertically by a factor of 3 (steepen)