



**Mathematical Sciences 106**  
**Calculus Functional Transformations**  
 August 29, 2011

**To graph**

**Vertical shifts**

$y = f(x) + k, k > 0$

$y = f(x) - k, k > 0$

**Horizontal shifts**

$y = f(x + h), h > 0$

$y = f(x - h), h > 0$

**Compressing or stretching**

$y = af(x), a > 0$

$y = f(ax), a > 0$

**Reflection about the  $x$ -axis**

$y = -f(x)$

**Reflection about the  $y$ -axis**

$y = f(-x)$

**Draw the Graph of  $f(x)$  and:**

Raise the graph of  $f$  by  $k$  units

Lower the graph of  $f$  by  $k$  units

Shift the graph of  $f$  to the left by  $h$  units

Shift the graph of  $f$  to the right by  $h$  units

Multiply each coordinate of  $y = f(x)$  by  $a$   
 Stretch the graph vertically if  $a > 1$   
 Compress the graph of  $f(x)$  vertically if  $0 < a < 1$

Multiply each coordinate of  $y = f(x)$  by  $\frac{1}{a}$   
 Stretch the graph horizontally if  $0 < a < 1$   
 Compress the graph of  $f(x)$  horizontally if  $a > 1$ .

Reflect the graph of  $f(x)$  about the  $x$ -axis

Reflect the graph of  $f(x)$  about the  $y$ -axis

**Functional Change to  $f$**

Add  $k$  to  $f(x)$

Subtract  $k$  from  $f(x)$

Replace  $x$  with  $x + h$

Replace  $x$  with  $x - h$

Multiply  $f(x)$  by  $a$

Replace  $x$  with  $ax$

Multiply  $f(x)$  by  $-1$ .

Replace  $x$  with  $-x$ .