Clemson Analysis Seminar Archive

Spring 2022

Necessary Conditions for Two Weight Weak Type Norm Inequalities for Multilinear Singular Integral Operators

Speaker: John-Oliver MacLellan (University of Alabama)

Date: January 28, 2022

Abstract: A central problem in harmonic analysis is to characterize the pairs of weights (u, v) so that a Calderón Zygmund operator maps $L^p(v) \to L^p(u)$. In this talk we will discuss necessary conditions for a multilinear Calderón Zygmund operator T to satisfy two weight weak type norm inequalities provided the kernel of T satisfies a weak non degeneracy condition. We generalize results from our recent paper, and earlier results from Lacey, Sawyer, and Uriarte-Tuero, and by Stein in the linear case. As an application of our techniques, we will show that in general a multilinear Calderón Zygmund does not satisfy a two-weight strong endpoint estimate.

The C^*-algebra generated by Toeplitz operators with quasi-radial symbol

Speaker: Vishwa Dewage (Louisiana State University)

Date: February 4, 2022

Abstract: In this talk we discuss Toeplitz operators with k-quasi-radial symbols acting on the Fock space $\mathcal{F}(\mathbb{C}^n)$. Toeplitz operators with k-quasi-radial symbols generate a commutative C*-algebra that is isometrically isomorphic to $C_{b,u}(\mathbb{N}_0^k)$ of bounded functions on $\mathbb{N}_0^k$ that are uniformly continuous with respect to the square root metric. In fact, the spectral functions (multi-sequences of eigenvalues) of these Toeplitz operators are dense in the space $C_{b,u}(\mathbb{N}_0^k)$. This talk is based on a joint work with Prof. Gestur Olafsson.

On the boundedness of oscillating singular integrals

Speaker: Duván Cardona Sánchez (Ghent University)

Date: February 11, 2022

Abstract: It was proved by Fefferman [1] and Fefferman and Stein [2], the weak $(1, 1)$ boundedness of oscillating singular integrals on $\mathbb{R}^n$, and the boundedness from the Hardy space H^1 into L^1, respectively. The aim of this talk is to discuss the recent extension of these results in the Euclidean setting (in view of the paradigm introduced by Grafakos and Stockdale in [3]) and on Lie groups of polynomial growth. In view of the solution of the Rockland conjecture by Helffer and Nourrigat [4], and of the Hormander theorem of sums of squares [5], our criteria are presented in terms of the analysis of sub-Laplacians and of Rockland operators. This talk is based on my joint works [CR1,CR2] with Michael Ruzhansky on the subject.

Space-time sampling for the functions with bounded spectrum

Speaker: Ilia Zlotnikov (University of Stavanger)

Date: Februrary 28, 2022

Abstract: Let $I = [a, b]$, $0 < a < b < \infty$ and let $\varphi_{u_{u\in I}}$ be a collection of functions that satisfy some natural assumptions (e.g., one may think that $\varphi_u(x) = e^{-ux^2}$). Assume that Λ is a uniformly discrete set. We will discuss what assumptions should be imposed on the set Λ to provide a stable reconstruction of the function (signal) f belonging to the Paley-Wiener space PW^p_σ or Bernstein space B_σ from the samples $\{f * \varphi_u(\lambda)\}_{\lambda\in\Lambda}$, i.e. to ensure the estimates

$$\|f\|_{L^\infty} \le K \sup_{u\in I,\lambda\in\Lambda} |f * \varphi_u(\lambda)|, \quad f \in B_\sigma,$$

or

$$D_1\|f\|^p_{L^p} \le \sum_{\lambda\in\Lambda} \int_I |f * \varphi_u(\lambda)|^p \, du \le D_2\|f\|^p_{L^p}, \quad f \in \mathrm{PW}^p_\sigma,$$

with constants K, D_1 and D_2 independent of f. This problem has several practical applications. For instance, we provide conditions for stable recovery of the initial data of the heat equation from given space–time samples.

References: [1] A. Ulanovskii, I. Zlotnikov, Reconstruction of bandlimited functions from space–time samples, J. Funct. Anal. 280(9), 108962 (2021).
[2] I. Zlotnikov, On planar sampling with Gaussian kernel in spaces of bandlimited functions, preprint: http://arxiv.org/abs/2104.09573

Characterizations of the Toeplitz algebra on the Fock space

Speaker: Raffael Hagger (Christian Albrechts University in Kiel)

Date: March 9, 2022

Abstract: Let μ_α denote the Gaussian measure on $\mathbb{C}^n$ given by

$$d\mu_\alpha(z) = \left(\frac{\alpha}{\pi}\right)^n e^{-\alpha|z|^2} \, dz$$

and consider the corresponding L^p-space $L^p_\alpha := L^p(\mathbb{C}^n, \mu_{\alpha p/2})$ for $p \in (1, \infty)$. The closed subspace of entire functions in L^p_α is called the Segal–Bargmann or Fock space and denoted by F^p_α. The corresponding orthogonal projection $P_\alpha : L^2_\alpha \to F^2_\alpha$ can be extended to a projection from L^p_α onto F^p_α. Of particular interest in this context are the Toeplitz operators $T_f : F^p_\alpha \to F^p_\alpha$, which are given by

$$T_f g := P_\alpha(fg)$$

for bounded symbols $f : \mathbb{C}^n \to \mathbb{C}$, and the Banach algebra generated by all Toeplitz operators with bounded symbol. For instance, a bounded linear operator on the Fock space is compact if and only if it is in the Toeplitz algebra and its so-called Berezin transform vanishes at infinity [1]. However, it is notoriously difficult to check directly whether a given operator is contained in the Toeplitz algebra. Recently, Fulsche [2] showed quite intriguingly that methods from quantum harmonic analysis can be used to study the Toeplitz algebra. In this talk I will present some direct consequences of Fulsche's work including several characterizations of the Toeplitz algebra which provide better membership criteria [3].

On Asymptotic Moments of Patterned Random Matrices

Speaker: Tapesh Yadav (University of Florida)

Date: March 18, 2022

Abstract: For a sufficiently nice 2 dimensional shape, we define its approximating matrix (or patterned matrix) as a random matrix with iid entries arranged according to the given pattern. For large approximating matrices, we observe that the eigenvalues roughly follow an underlying distribution. This phenomenon is similar to the classical observation on Wigner matrices. We prove that the moments of such matrices converge asymptotically as the size increases and equals to the integral of a combinatorially-defined function which counts certain paths on a finite grid.

Quantitative estimates in the matrix weighted setting

Speaker: Israel Rivera-Ríos (University of Málaga)

Date: April 1, 2022

Abstract: In this talk we will provide an overview from the quantitative theory of Muckenhoupt weights to the quantitative matrix weights theory. The plan of the talk will consist in revisiting sparse domination ideas, which have been a fruitful field of development in the scalar setting, and providing some insight on how they have been adapted to the vector valued setting, making them useful to obtain quantitative weighted estimates in that setting.

Integral geometry problems in Lorentzian geometry

Speaker: Yiran Wang (Emory University)

Date: April 8, 2022

Abstract: We consider the light ray transform on Lorentzian manifolds, which concerns the integral of functions along light-like (or null) geodesics. The transform is related to the Radon transform or geodesic ray transform in the Riemannian setting. An outstanding question is what information regarding the function can be recovered from the light ray transform. In this talk, we discuss recent developments, including injectivity, stability results and microlocal properties of the transform. Also, we discuss the application to some inverse problems in cosmology. In particular, we will show how to recover space-time structures from the Cosmic Microwave Background (CMB).

Kohler-Jobin meets Ehrhard: the sharp lower bound for the Gaussian principal frequency while the Gaussian torsional rigidty is fixed, via rearrangements

Speaker: Orli Herscovici (Georgia Institute of Technology)

Date: April 15, 2022

Abstract: In this talk, we show an adaptation of the Kohler-Jobin rearrangement technique to the setting of the Gauss space. As a result, we present the Gaussian analogue of the Kohler-Jobin's resolution of a conjecture of Polya-Szego: when the Gaussian torsional rigidity of a (convex) domain is fixed, the Gaussian principal frequency is minimized for the half-space. At the core of this rearrangement technique is the idea of considering a "modified" torsional rigidity, with respect to a given function, and rearranging its layers to half-spaces, in a particular way; the Rayleigh quotient decreases with this procedure.

We emphasize that the analogy of the Gaussian case with the Lebesgue case is not to be expected here, as in addition to some soft symmetrization ideas, the argument relies on the properties of some special functions; the fact that this analogy does hold is somewhat of a miracle. Based on joint work with Galyna Livshyts.

Inversion, convexity, and realization problems in free noncommutative analysis

Speaker: Mark Mancuso (Lafayette College)

Date: April 25, 2022

Abstract: This talk will provide an overview of some central problems in free analysis related to noncommutative inversion, matrix and operator convexity, and obtaining realizations. Free analysis, or noncommutative function theory, finds its origin in J. L. Taylor's work on the functional calculus of several noncommuting operators. Loosely speaking, noncommutative function theory can be thought of as an analogue of several complex variables where the holomorphic functions act on domains consisting of either tuples of bounded operators on a Hilbert space or tuples of square complex matrices of all sizes. The simplest examples of noncommutative functions are given by free polynomials with complex coefficients; $p(X, Y) = XY^2 + XY - YX$ is a free polynomial and can be evaluated at pairs of matrices or operators. Well-known results along with recent progress will be shared throughout this talk.

Fall 2021

Alpha-area Siegel-Veech constants for branched cyclic covers

Speaker: Martin Schmoll (Clemson University)

Date: September 8, 2021

Abstract: We will present a relative formula for asymptotic quadratic growth rates of periodic cylinders for branched cyclic covers that seems to hold in more generality. We will talk about to which extend we can prove this formula and its inner structure. Background on flat surfaces is provided (mostly informally), as well as a brief introduction into the Siegel-Veech formula and its evaluation. An interesting feature of the formula for general surfaces of higher genus is that it is essentially the formula for branched torus covers. It is worth noting that the resulting formula has been predicted by computer experiments using an approach that does not involve the Siegel-Veech formula our proof relies on.

This is ongoing research together with David Aulicino (Brooklyn College), Aaron Calderon (Yale) and Nick Salter (Notre Dame).

A completeness and cyclicty problem for a semigroup of weighted composition operator

Speaker: Chris Felder (Washington University in St. Louis)

Date: September 17, 2021

Abstract: This talk will be largely expository, following recent work of S. Waleed Noor. We will discuss a completeness problem in the classical Hardy space, along with a cyclicity problem for a semigroup of weighted composition operators. We will then talk about a least-squares approach to solving such problems.

On the John-Nirenberg constant of BMO^p, $0 < p < 1$

Speaker: Brandon Sweeting (University of Alabama)

Date: September 24, 2021

Abstract: We present sharp L^p lower bounds for logarithms of A_∞ weights as a means of estimating the John-Nirenberg constant of the space BMO^p, $0 < p < 1$. The corresponding Bellman function solves the homogeneous Monge-Ampère equation, but the geometry of the solution goes beyond established theory due to the lack of regularity in the boundary condition. This is joint work with Leonid Slavin.

Stabilization rates for the damped wave equation with polynomial and oscillatory damping

Speaker: Perry Kleinhenz (Michigan State University)

Date: September 29, 2021

Abstract: In this talk I will discuss energy decay of solutions of the Damped wave equation. After giving an overview of classical results I'll focus on the torus with damping that does not satisfy the geometric control condition. In this setup properties of the damping at the boundary of its support determine the decay rate, however a general sharp rate is not known.

I will discuss damping which is 0 on a strip and vanishes either like a polynomial x^b or an oscillating exponential $e^{-1/x}\sin^2(1/x)$. Polynomial damping produces decay of the semigroup at exactly $t^{-(b+2)/(b+3)}$, while oscillating damping produces decay at least as fast as $t^{-4/5+\delta}$ for any $\delta > 0$. I will explain how these model cases are proved and how they direct further study of the general sharp rate.

An approach to universality using Weyl m-functions

Speaker: Milivoje Lukić (Rice University)

Date: October 8, 2021

Abstract: In this talk, I will present joint work with Benjamin Eichinger and Brian Simanek: a new approach to universality limits for orthogonal polynomials on the real line which is completely local and uses only the boundary behavior of the Weyl m-function at the point. We show that bulk universality of the Christoffel–Darboux kernel holds for any point where the imaginary part of the m-function has a positive finite nontangential limit. This approach is based on studying a matrix version of the Christoffel–Darboux kernel and the realization that bulk universality for this kernel at a point is equivalent to the fact that the corresponding m-function has normal limits at the same point. Our approach automatically applies to other self-adjoint systems with 2×2 transfer matrices such as continuum Schrodinger and Dirac operators. We also obtain analogous results for orthogonal polynomials on the unit circle.

Random Interpolating Sequences in the Polydisc and the Unit Ball

Speaker: Alberto Dayan (Norwegian University of Science and Technology)

Date: October 15, 2021

Abstract: A random sequence Z in the unit disc is determined by a sequence of deterministic radii and a sequence of i.i.d. random variables uniformly distributed on the unit circle. Chochran and Rudowicz found the $0-1$ Kolmogorov law for Z to be interpolating, that is, the cut-off condition on the a-priori fixed radii in order for Z to be interpolating almost surely. In this talk, we will extend their work to random interpolating sequences for bounded analytic functions in the d-dimensional polydisc and for the Besov-Sobolev spaces on the unit ball. The case of the Besov-Sobolev spaces is more treatable, since such spaces have their interpolating sequences well understood and characterized in the deterministic setting. This is not the case for interpolating sequences in the polydisc: in this second case our necessary and sufficient conditions for almost sure interpolation do not coincide, and they are obtained by looking at the decay of related random Grammians off their diagonals.

This is a joint work with Brett Wick and Shengkun Wu

Weighted Inequalities on Spaces of Homogeneous Type

Speaker: Manasa Vempati (Georgia Institute of Technology)

Date: October 27, 2021

Abstract: We will discuss the two weight inequalities for Calderón-Zygmund operators and commutators. We work in the setting of spaces of homogeneous type defined in the sense of Coifman and Weiss. Subject to the pair of weights u and v satisfying a side condition, we will show a characterization for the boundedness of a Calderon-Zygmund operator T from $L^2(u)$ to $L^2(v)$ in terms of the A_2 condition and two testing conditions. We will also give the two weight quantitative estimates for the commutator of maximal functions and the maximal commutators with respect to the symbol in weighted BMO space on spaces of homogeneous type.

A wavepacket approach to the random Schrodinger equation

Speaker: Felipe Hernandez (Stanford University)

Date: November 1, 2021

Abstract:I will discuss my work in progress on the Schrodinger equation perturbed by a random potential. This equation is a simple model for wave propagation in random environments. A key feature of solutions to this equation is a diffusion phenomenon, meaning that the mass of the wavefunctions weakly solves a heat equation. A previous work of Erdos, Salmhofer, and Yau in 2007 derived this heat equation using a complex diagrammatic expansion. I will explain in this talk how wavepacket decompositions can be used to give a more geometric understanding of the problem.

A strange limit of horocycle ergodic measures in a stratum of translation surfaces

Speaker: Jon Chaika (University of Utah)

Date: November 12, 2021

Abstract: The main result of this talk is that in the space of unit area translation surfaces with one cone point there is a weak-star limit of measures on periodic horocycles that is fully supported in the 7-dimensional space but gives positive measure to a 3-dimensional submanifold. As a consequence we obtain a non-genericity result for the horocycle flow in this space. I will define the terminology. This is joint work with Osama Khalil and John Smillie.

Pressure Function on Compact Symbolic Systems

Speaker: Tamara Kucherenko (CUNY)

Date: November 19, 2021

Abstract: We study the flexibility of the pressure function of a continuous potential (observable) with respect to a parameter regarded as the inverse temperature. The points of non-differentiability of this function are of particular interest in statistical physics, since they correspond to phase transitions. It is well known that the pressure function is convex, Lipschitz, and has an asymptote at infinity. We prove that in a setting of one-dimensional compact symbolic systems these are the only restrictions. We present a method to explicitly construct a continuous potential whose pressure function coincides with any prescribed convex Lipschitz asymptotically linear function starting at a given positive value of the parameter. In fact, we establish a multidimensional version of this result. As a consequence, we obtain that for a continuous observable the phase transitions can occur at a countable dense set of temperature values. We go further and show that one can vary the cardinality of the set of ergodic equilibrium states as a function of the parameter to be any number, finite or infinite.

Optimal Tomography with Local Data

Speaker: Francis Chung (University of Kentucky)

Date: November 29, 2021

Abstract: Optical tomography is the problem of reconstructing interior properties of an object from optical measurements at the boundary. The mathematical version of this problem is to reconstruct the coefficients of a PDE (in my case, a radiative transport equation) from measurements of the solutions at the boundary. In this talk I'll discuss an optical tomography problem with local data, in which measurements are restricted to a subset of the boundary. I plan to introduce the problem, discuss the wider context, and describe a recent result.

Spring 2021

Translational tilings: structure and decidability

Speaker: Rachel Greenfeld (University of California Los Angeles)

Date: January 13, 2021

Abstract: Let F be a finite subset of the d-dimensional integer lattice. We say that F is a translational tile of $\mathbb{Z}^d$ if it is possible to cover $\mathbb{Z}^d$ by translates of F with no overlaps. Given a finite subset F of $\mathbb{Z}^d$, could we determine whether F is a translational tile in finite time? Suppose that F does tile, what can be said about the structure of the tiling? A well known argument of Wang shows that these two questions are closely related. In the talk, I will introduce and demonstrate this relation and present some new results, joint with Terence Tao, on the rigidity of tiling structures in $\mathbb{Z}^2$, and their applications to decidability.

Thin elastic plates involving fractional rotational forces: semigroups regularity and stability

Speaker: Louis Tebou (Florida International University)

Date: January 20, 2021

Abstract: In this talk, finite thin elastic plates involving fractional rotational forces are considered. Using resolvent estimates, new regularity and stability results for the underlying semigroups are established. First, I examine a thermoelastic plate with fractional rotational forces, and prove Gevrey regularity, as well as exponential stability of the associated semigroup. Afterward, I analyze a mechanically damped plate, where the rotational forces and damping involve fractional powers of the Laplacian. For this latter model, it will be shown that the underlying semigroup is analytic, or of a certain Gevrey class, is exponentially or polynomially stable, depending on the relationship between the fractional exponents of the damping and rotational forces. The models considered are new and lie between the classical Euler-Bernoulli model and the Kirchhoff model for thin plates in either case. This is a joint-work with my colleagues Valentin Keyantuo (University of Puerto Rico, San Juan) and Mahamadi Warma (George Mason University)

Strongly Peaking Representations and Compressions of Operator Systems

Speaker: Benjamin Passer (United States Naval Academy)

Date: January 27, 2021

Abstract: (Joint work with Ken Davidson.) Just as a compact convex set is generated by its extreme points, it is known that an operator system is completely normed by its boundary representations. We analyze a special class of boundary representations in order to study operator systems which are presented in a smallest possible way. Our results extend recent work in the study of free spectrahedra and matrix convex sets.

Uncertainty principle in harmonic analysis and its applications

Speaker: Mishko Mitkovski (Clemson University)

Date: February 3, 2021

Abstract: I will present several new forms of the uncertainty principle. These new forms can be viewed as extensions/sharpening of some classical uncertainty principles, in a sense that we impose restrictions on the Fourier support and deduce sampling inequalities. Many

of our results were inspired by some control and damping problems for linear PDE's and I will try to present some of these applications. This is a collaborative work with W. Green, B. Jaye, and H. Li.

Data-Dependent Distances for Unsupervised and Active Learning

Speaker: James Murphy (Tufts University)

Date: February 10, 2021

Abstract: Approaches to unsupervised clustering and active learning with data-dependent distances are proposed. By considering metrics derived from data-driven graphs, robustness to noise and class geometry is achieved. The proposed algorithms enjoy theoretical guarantees on flexible data models, and also have quasilinear computational complexity in the number of data points. Connections will be made to geometric analysis and percolation theory. Applications to image processing and biological networks will be shown, demonstrating state-of-the-art empirical performance.

Weighted inequalities for Haar multipliers

Speaker: Claire Huang (Washington University in St. Louis)

Date: February 19, 2021

Abstract: A measure on $\mathbb{R}^n$ is called "dyadic doubling" if the measure ratio of any dyadic parent and its dyadic child is uniformly bounded. This property enables a stopping time argument presented by Katz and Pereyra in their 1998 survey on the L^p boundedness of Haar multipliers. Highly inspired by these two authors' work, this presentation focuses on extending their results to weighted spaces. In particular, we are interested in the L^p boundedness of Haar multipliers with respect to weights in dyadic Muckenhoupt classes, as they guarantee dyadic doubling, and the closely related weights in dyadic reverse Hölder classes.

Trace minmax functions and the radical Laguerre-Polya class

Speaker: James Pascoe (University of Florida)

Date: February 22, 2021

Abstract: We classify functions $f : (a, b) \to \mathbb{R}$ which satisfy the inequality $\operatorname{tr} f(A) + f(C) \geq \operatorname{tr} f(B) + f(D)$ when $A \leq B \leq C$ are self-adjoint matrices, $D = A + C - B$, the so-called trace minmax functions. (Here $A \leq B$ if $B - A$ is positive semidefinite, and f is evaluated via the functional calculus.) A function is trace minmax if and only if its derivative analytically continues to a self map of the upper half plane. The negative exponential of a trace minmax function $g = e^{-f}$ satisfies the inequality $\det g(A)g(C) \leq \det g(B)g(D)$ for A, B, C, D as above. We call such functions determinant isoperimetric. We show that determinant isoperimetric functions are in the "radical" of the the Laguerre-Polya class. We derive an integral representation for such functions which is essentially a continuous version of the Hadamard factorization for functions in the the Laguerre-Polya class. We apply our results to give some equivalent formulations of the Riemann hypothesis. Finally, we discuss some recent joint work with Kelly Bickel and Meredith Sargent on relaxations of the above framework, and an avenue for obtaining zero-free regions for the Riemann zeta function.

On dual molecules and convolution-dominated operators

Speaker: Jordy Timo van Velthoven (University of Vienna)

Date: March 3, 2021

Abstract: This talk considers coherent systems arising from unitary representations. It is shown that frames and Riesz sequences can be obtained whose dual systems form molecules, ensuring that their elements satisfy appropriate size estimates as if they were obtained one from the other by the group action. As a consequence, the canonical expansions extend to associated Banach spaces. The main tool is a local holomorphic calculus for convolution-dominated operators, valid for groups with possibly exponential volume growth.

Control and Inverse Problems for a Strongly Coupled Hyperbolic System of PDEs

Speaker: Jason Kurz (Clemson University)

Date: March 12, 2021

Abstract: In this presentation we consider an inverse and control problem for the Mindlin–Timoshenko plate system, which is a strongly coupled two dimensional system consisting of a wave equation and a system of isotropic elasticity, that arises in modeling plate vibrations especially at high frequencies and thicker plates. More precisely, we prove the global uniqueness of recovering the plate density from a single boundary measurement under appropriate geometrical assumptions. Secondly, we demonstrate the controllability of the system via an indirect control technique that proves a two-level indirect inverse observability estimate. Our approach for both problems relies on diagonalizing the principal part of the system and making it a system of wave-like equations.

Wavelet Representation of Singular Integral Operators

Speaker: Tyler Williams (Washington University in St. Louis)

Date: March 26, 2021

Abstract: We describe a new representation technique for one and multiple parameter singular integrals in terms of continuous model operators. Unlike the well established dyadic counterpart, our representation reflects the additional kernel smoothness of the operator being analyzed. Our representation formulas lead naturally to a new family of $T(1)$ theorems on weighted Sobolev spaces whose smoothness index is naturally related to kernel smoothness. I present the one parameter case, where we obtain the Sobolev space analogue of the A_2 theorem; that is, sharp dependence of the Sobolev norm of T on the weight characteristic is obtained in the full range of exponents. In the bi-parametric setting, where we obtain quantitative A_p estimates which are best known, and sharp in the range $maxp, p' \geq 3$. These estimates are beyond the reach of current dyadic methods.

Boundedness of commutators on weighted Hardy spaces

Speaker: Marie-Jose Kuffner (Johns Hopkins University)

Date: April 2, 2021

Abstract: It is known that boundedness of the commutator $[b, H]$ on weighted L^p-spaces for $1 < p < \infty$ is characterized by b being in a certain BMO space adapted to the given weights. In this talk, we present the case $p = 1$ and discuss the space that characterizes boundedness of $[b, H]$ on the weighted Hardy space $H^1(w)$ for certain A_p weights.

Inverse Problems for Nonlinear PDEs

Speaker: Ting Zhou (Northeastern University)

Date: April 7, 2021

Abstract: In this talk, I will demonstrate the higher order linearization approach to solve several inverse boundary value problems for nonlinear PDEs modeling nonlinear electromagnetic optics including nonlinear time-harmonic Maxwell's equations with Kerr-type and second harmonic generation nonlinearity. The problem will be reduced to solving for the coefficient functions from their integrals against multiple linear solutions. We will focus our discussion on different choices of linear solutions. A similar problem for nonlinear magnetic Schrodinger equation will be considered as a comparison.

Tiling the integers with translates of one tile: the Coven-Meyerowitz tiling conditions for three prime factors

Speaker: Itay Londner (University of British Columbia)

Date: April 12, 2021

Abstract: It is well known that if a finite set of integers A tiles the integers by translations, then the translation set must be periodic, so that the tiling is equivalent to a factorization $A + B = \mathbb{Z}_M$ of a finite cyclic group. Coven and Meyerowitz (1998) proved that when the tiling period M has at most two distinct prime factors, each of the sets A and B can be replaced by a highly ordered "standard" tiling complement. It is not known whether this behavior persists for all tilings with no restrictions on the number of prime factors of M.

In an ongoing collaborations with Izabella Laba, we proved that this is true when $M = (pqr)^2$. In my talk I will discuss this problem and introduce the main ingredients of the proof.

Extrapolation of compactness on weighted spaces

Speaker: Stefanos Lappas (University of Helsinki)

Date: April 21, 2021

Abstract: The extrapolation theorem of Rubio de Francia is one of the most powerful tools in the theory of weighted norm inequalities: it allows one to deduce an inequality (often but not necessarily: the bounded of an operator) on all weighted L^p spaces with a range of p, by checking it just for one exponent p (but all relevant weights). My topic is an analogous method for extrapolation of compactness. In a relatively soft way, it recovers several recent results about compactness of operators on weighted spaces and also gives some new ones. This is a joint-work with Tuomas Hytönen.

Fall 2020

C-cyclical monotonicity in optimal transport and beyond

Speaker: Mathias Beiglböck (University of Vienna)

Date: September 9, 2020

Abstract: A fundamental idea in optimal transport is that the optimality of a transport plan is reflected by the geometry of its support set. Often this is key to understanding the transport problem. On the level of support sets, the relevant notion is c-cyclical monotonicity. The relevance of this concept for the theory of optimal transport has been fully recognized by Gangbo and McCann, based on earlier work of Knott and Smith and Ruschendorf among others. Since then it has been understood through the work of various authors that analogous "monotonicity principles" are also highly useful in a number of related areas. The goal of this talk is to provide an introduction to these ideas and to sketch some of the recent developments.

Weighted Estimates for the Bergman and Szego Projections on Strongly Pseudoconvex Domains with Near Minimal Smoothness

Speaker: Nathan Wagner (Washington University in St. Louis)

Date: September 23, 2020

Abstract: The Bergman and Szego projections are fundamental operators in complex analysis in one and several complex variables. Consequently, the mapping properties of these operators on L^p and other function spaces have been extensively studied. In this talk, we discuss some recent results for these operators on strongly pseudoconvex domains with near minimal smoothness. In particular, weighted L^p estimates are obtained, where the weight belongs to a suitable generalization of the Bekolle-Bonami or Muckenhoupt class. For these domains with less boundary regularity, we use an operator-theoretic technique that goes back to Kerzman and Stein. We also obtain weighted estimates for the endpoint $p = 1$, including weak-type $(1, 1)$ estimates. Here we use a modified version of singular-integral theory and a generalization of the Riesz-Kolmogorov characterization of precompact subsets of Lebesgue spaces. This talk is based on joint work with Brett Wick and Cody Stockdale.

What is a quantum Markov chain? A beginner's look into non-commutative probability

Speaker: Renato Feres (Washington University in St. Louis)

Date: October 7, 2020

Abstract: For a mathematician, basic quantum theory may be viewed as a generalized (non-commutative) probability theory. The current mathematical research in non-commutative probability and stochastic processes has been greatly influenced by scientific developments in quantum communication and the physics of open quantum systems. In this presentation, I wish to explore a few of the more fundamental ideas in (discrete time,Markovian) quantum stochastic processes, guided by a class of classical Markov chains obtained from elementary mechanical systems.

A Calderón-Zygmund decomposition for von Neumann algebra valued functions

Speaker: José Conde-Alonso Universidad Autónoma de Madrid

Date: October 14, 2020

Abstract: The classical Calderón-Zygmund decomposition is a fundamental tool that helps one study endpoint estimates near L^1. In this talk, we shall study an extension of the decomposition to a particular operator valued setting where noncommutativity makes its appearance, allowing to get rid of the (usually necessary) UMD property of the Banach space where functions take values.

A two-dimensional inverse problem in magnetism

Speaker: Elodie Pozzi (St. Louis University)

Date: October 21, 2020

Abstract: Inverse problems have known a recent development in many fields like signal processing, medical imaging and more recently paleomagnetism. Broadly speaking, an inverse problem consists in reconstructing from a set of measurements the original source. We consider a two dimensional inverse problem in magnetism to estimate the net moment represented by the mean value of a function supported on an interval K of the real line from the partial knowledge of the magnetism on another interval S located on the parallel line to K at height h. We will see how this question can be rephrased using complex analysis, harmonic analysis and operator theory. To estimate the mean value, we will construct and solve a constrained approximation problem. This talk is based on a joint work with Juliette Leblond, INRIA, France.

Commutators and bounded mean oscillation

Speaker: Brett Wick (Washington University in St. Louis)

Date: October 28, 2020

Abstract: We will discuss some recent results about commutators of certain Calderón-Zygmund operators and BMO spaces and how these generate bounded operators on Lebesgue spaces. Results on the Heisenberg group, pseudoconvex domains with C^2 boundary, and other examples will be explained. This talk is based on joint collaborative work.

A different approach to endpoint weak-type estimates for Calderón-Zygmund operators

Speaker: Cody Stockdale (Clemson University)

Date: November 4, 2020

Abstract: The weak-type $(1, 1)$ estimate for Calderón-Zygmund operators is fundamental in harmonic analysis. We investigate weak-type inequalities for Calderón-Zygmund singular integral operators using the Calderón-Zygmund decomposition and ideas inspired by Nazarov, Treil, and Volberg. We discuss applications of these techniques in the Euclidean setting, in weighted settings, for multilinear operators, for operators with weakened smoothness assumptions, and in studying the dimensional dependence of the Riesz transforms.

Sharp trace inequalities involving differential operators

Speaker: Franz Gmeineder (University of Bonn)

Date: November 11, 2020

Abstract: Essentially by the non-availability of singular integrals on L^1, one cannot estimate the L^1-norms of the full k^{th} order gradients against those of k^{th} order differential expressions in general. This is known as Ornstein's Non-Inequality. Over the past years, however, a theory has been developed that allows to estimate lower order quantities by the L^1-norms of differential expressions in a sharp way. In this talk, which is joint work with Lars Diening (Bielefeld), we present sharp conditions on differential operators to retrieve the well-known boundary trace estimates from the Sobolev case.

Sampling, density and equidistribution

Speaker: José Luis Romero (University of Vienna)

Date: November 17, 2020

Abstract: The sampling problem concerns the reconstruction of every function within a given class from their values observed only at certain points (samples). A density theorem gives necessary or sufficient conditions for such reconstruction in terms of an adequate notion of density of the set of samples. The most classical density theorems, due to Shannon and Beurling, involve bandlimited functions (that is, functions whose Fourier transforms are supported on the unit interval) and provide a precise geometric characterization of all configurations of points that lead to reconstruction. I will present modern variants of these results and their applications in other fields of analysis.

Spectral geometry of CR manifolds

Speaker: Yunus Zeytuncu (University of Michigan Dearborn)

Date: November 23, 2020

Abstract: In this talk, we look at the spectrum of the Kohn Laplacian on spheres $\mathbb{S}^{2n-1}$ and the Rossi sphere $\mathbb{S}^3_t$. We relate spectral calculations to the geometry of the manifolds. In the first part, we use the lower bounds on the essential spectrum to understand the embeddability of the Rossi sphere. In the second part, we compute the leading coefficient in the asymptotic expansion of the counting function on $\mathbb{S}^{2n-1}$ by a Tauberian argument, and we relate it to the volume of the spheres.

A unified method for commutators of Calderón-Zygmund operators

Speaker: Kabe Moen (University of Alabama)

Date: December 2, 2020

Abstract: We present a unified method to obtain weighted estimates of linear and multilinear commutators with BMO functions, that is amenable to a plethora of operators and functional settings. Our approach elaborates on a commonly used Cauchy integral trick, recovering many known results but yielding also numerous new ones. We also look at some new two weight bump conditions for commutators.