

PALmetto Number Theory Series XXIX

SCHEDULE OF ACTIVITIES

Talks will take place in Brackett Hall Rooms 100, 111, 113-114.

Saturday, December 2, 2017

- 9:00 - 9:30 Coffee and other refreshments
- 9:30 - 9:50 **Shenhui Liu** (University of Toronto), *Nonvanishing of central L -values of $GL(3)$ Maass forms*
- 9:55 - 10:15 **Huixi Li** (Clemson University), *Sun's Conjecture on Symmetric Quadratic Forms*
- 10:20 - 10:40 **Lea Beneish** (Emory University), *Weight two moonshine*
- 10:40 - 11:00 Break
- 11:00 - 12:00 **Henri Darmon** (McGill University), *p -adic analysis and Hilbert's twelfth problem*
- 12:00 - 1:40 Lunch
- 1:40 - 2:40 **Cristian Popescu** (University of California - San Diego), *On a conjecture of Gross on special values of L -functions*
- 2:40 - 3:00 Break
- 3:00 - 3:20 **Kyle Pratt** (University of Illinois), *A lower bound for the least prime in an arithmetic progression*
- 3:25 - 3:45 **Wanlin Li** (University of Wisconsin), *Vanishing of hyperelliptic L -functions at the central point*
- 3:45 - 4:00 Break
- 4:00 - 4:45 **Isabel Leal**, (University of Chicago), *Generalized Hasse-Herbrand ψ -functions*

Sunday, December 3, 2017

- 9:00 - 9:30 Coffee and other refreshments
- 9:30 - 9:50 **Stevo Bozinovski** (South Carolina State University), *An absolute value theorem for Riemann zeta function*
- 9:55 - 10:15 **Malcolm Rupert** (Clemson University), *On the local theta lift from $\mathrm{GSO}(4)$ to $\mathrm{GSp}(4)$*
- 10:15 - 10:30 Break
- 10:30 - 11:30 **Jennifer Balakrishnan** (Boston University), *Quadratic Chabauty*
- 11:30 - 11:45 Break
- 12:00-12:20 **Solly Parenti** (University of Wisconsin), *The Colmez Conjecture*
- 12:25 - 12:45 **Jackson Morrow** (Emory University), *Irrational points on random hyperelliptic curves*

The organizers thank the National Science Foundation, and Department of Mathematical Sciences at Clemson University for their support.

Abstracts

JENNIFER BALAKRISHNAN, Boston Univeristy, *Quadratic Chabauty*

I will describe how p-adic heights can be used to find rational points on certain curves of genus at least 2, in the spirit of Kim's nonabelian Chabauty program. This is based on joint work with Netan Dogra, Steffen Mueller, Jan Tuitman, and Jan Vonk.

Sunday, 10:30 - 11:30

LEA BENEISH, Emory University, *Weight two moonshine*

This research has two main objectives: one is to investigate higher weight moonshine, and the other is to put umbral moonshine in a context closer to monstrous moonshine. We take Mathieu moonshine as a starting point, giving a construction which associates weakly holomorphic modular forms of weight 2 to elements of M_{23} . We prove the existence of a corresponding graded M_{23} -module.

Saturday, 10:20 - 10:40

STEVO BOZINOVSKI, South Carolina State University, *An absolute value theorem for Riemann zeta function*

The following result is obtained: $Riemann_zeta(-1) = |Dirichlet_eta(-2)|$ A discussion will be given on the consequence of this result for the theory of divergent series and the number theory. This is a joint work with Adrijan Bozinovski.

Sunday, 9:30 - 9:50

HENRI DARMON, McGill University, *p-adic analysis and Hilberts twelfth problem*

Modular functions play an important role in many aspects of number theory. The theory of complex multiplication, one of the grand achievements of the subject in the 19th century, asserts that the values of modular functions at quadratic imaginary arguments generate (essentially all) abelian extensions of imaginary quadratic fields. Hilbert's twelfth problem concerns the generalization of this theory to other base fields. I will describe an ongoing work in collaboration with Jan Vonk which identifies a class of functions that seem to play the role of modular functions for real quadratic fields. A key difference with the classical setting is that they are meromorphic functions of a p -adic variable (defined in the framework of "rigid analysis" introduced by Tate) rather than of a complex variable.

Saturday, 11:00 - 12:00

ISABEL LEAL, University of Chicago, *Generalized Hasse-Herbrand ψ -functions*

The classical Hasse-Herbrand ψ -function is an important object in ramification theory, related to higher ramification groups. In this talk, I will discuss generalizations of the Hasse-Herbrand function and go over some of their properties. These generalized ψ -functions are defined for extensions L/K of complete discrete valuation fields where the residue field k of K is perfect of characteristic $p > 0$ but the residue field l of L is possibly imperfect.

Saturday, 4:00 - 4:45

HUIXI LI, Clemson University, *Sun's Conjecture on Symmetric Quadratic Forms*

Fermat's theorem on sums of two squares states that an odd prime p can be written as the sum of two integer squares $a_p^2 + b_p^2$ if and only if p is congruent to 1 modulo 4. It is easy to see that such representation is unique if we require $a_p > b_p > 0$. In this talk I will compute the asymptotic of the ratio of the sum of the a_p terms and the sum of the b_p terms, which leads to a result that verifies a conjecture by Zhiwei Sun. This is joint work with Brown and James.

Saturday, 9:55 - 10:15

WANLIN LI, University of Wisconsin, *Vanishing of hyperelliptic L -functions at the central point*

We obtain a lower bound on the number of quadratic Dirichlet L-functions over the rational function field which vanish at the central point $s = 1/2$. This is in contrast with the situation over the rational numbers, where a conjecture of Chowla predicts there should be no such L-functions. The approach is based on the observation that vanishing at the central point can be interpreted geometrically, as the existence of a map to a fixed abelian variety from the hyperelliptic curve associated to the character.

Saturday, 3:25 - 3:45

SHENHUI LIU, University of Toronto, *Nonvanishing of central L-values of GL(3) Maass forms*

In this talk, we work with certain GL(3) L-functions at the central point of the critical strip. Specifically, consider an orthogonal basis $\{\phi_j\}$ of Hecke–Maass forms for $SL(3, \mathbb{Z})$. Using the method of moments and the mollification method, we obtain a positive-proportional nonvanishing result for $L(\frac{1}{2}, \phi_j)$ when the spectral parameters of ϕ_j are concentrated around a large parameter T . The main tool we employ is the GL(3) Kuznetsov trace formula. This is joint work with Bingrong Huang and Zhao Xu.

Saturday, 9:30 - 9:50

JACKSON MORROW, Emory University, *Irrational points on random hyperelliptic curves*

Let d and g be positive integers with $1 < d < g$. If d is odd, we prove there exists $B(d) > 0$ such that a positive proportion of odd genus g hyper elliptic curves over \mathbf{Q} have at most $B(d)$ points of degree d . If d is even, we similarly bound the degree d points not pulled back from degree $d/2$ points of the projective line. Our proof proceeds by refining Parks recent application of tropical geometry to symmetric power Chabauty, and then applying results of Bhargava and Gross on average ranks of Jacobians of hyperelliptic curves. This is joint work with Joseph Gunther.

Sunday, 12:25-12:45

SOLLY PARENTI, University of Wisconsin, *The Colmez Conjecture*

The Faltings height of an abelian variety is a fundamental invariant that was introduced in the proof of the Mordell conjecture. Pierre Colmez formulated a conjectural interpretation of the Faltings height of a CM abelian variety in terms of log derivatives of Artin L-functions coming from the data of the CM type. I will talk about this conjecture, what is known, and my recent work on the case where the CM abelian variety admits an action by an imaginary quadratic field.

Sunday, 12:00 - 12:20

CRISTIAN POPESCU, University of California - San Diego, *On a conjecture of Gross on special values of L-functions*

I will report on my recent work on an equivariant refinement of a conjecture of Gross linking special values of p -adic and global L -functions. I will make connections with recent results of Dasgupta, Kakde and Ventullo on the original, non-equivariant Gross conjecture.

Saturday, 1:40 - 2:40

KYLE PRATT, University of Illinois, *A lower bound for the least prime in an arithmetic progression*

Fix k a positive integer, and let ℓ be coprime to k . Let $p(k, \ell)$ denote the smallest prime in the residue class $\ell \pmod{k}$, and set $P(k)$ to be the maximum of $p(k, \ell)$ over residue classes ℓ .

In this talk I will describe joint work with Junxian Li and George Shakan, in which we showed that for almost every k one has $P(k) \gg \phi(k) \log k \log_2 k \log_4 k / \log_3 k$. This improves an earlier bound of Pomerance, and answers a question of Ford, Green, Konyagin, Maynard, and Tao. I will discuss some ideas used in the proof, which has roots in recent work on large gaps between primes. I will also discuss some heuristics about the size of $P(k)$.

Saturday, 3:00 - 3:20

MALCOLM RUPERT, Clemson University, *On the local theta lift from GSO(4) to GSp(4)*

There is a global Theta correspondence for the pair $(\mathrm{GO}(4), \mathrm{GSp}(4))$. If π is an irreducible automorphic representation produced from a Cuspidal Hilbert Modular form with Γ_0 level over a quadratic field, then $\Theta(\pi) = \Pi$ contains a Paramodular form. We discuss a realization of the local theta correspondence and a certain local-to-global theorem.

Sunday, 9:55 - 10:15