

MTHSC 102 SECTION 1.1 – FUNCTIONS: FOUR REPRESENTATIONS

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NOTE

We will typically represent related data in four ways or from four viewpoints.

- Numerically (using a chart or table of data)
- Graphically (using a scatter plot or continuous graph)
- Verbally (using a word description)
- Algebraically (using a mathematical model).

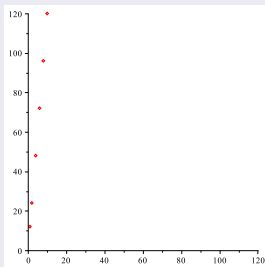
EXAMPLE

Suppose that we wish to fill in a low swampy area with 120 cubic yards of dirt. Suppose that we will use a truck which is capable of moving 12 cubic yards of soil per load and that we can deliver 1 load of soil per hour around the clock until the job is complete. The amount of soil moved after t hours is given by the following chart.

Elapsed time (hours)	Amount of Soil
1	12
2	24
4	48
6	72
8	96
10	120

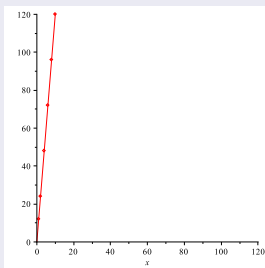
EXAMPLE CONTINUED...

We could also represent the above data by the following plot



EXAMPLE CONTINUED...

Or the following continuous graph



An algebraic description of the amount A of soil moved after t hours is given by

$$A(t) = 12t$$

DEFINITION

A function is a rule that assigns exactly one output value to each possible input value.

NOTE

The output value of f for the input x is denoted $f(x)$ and is read “ f of x ”.

EXAMPLE

If the price of gas is fixed at \$3.93 per gallon and g is the number of gallons that we wish to purchase, then the cost C will be a function of g , because for a given value of g , say $g = 10$ gallons there is only one possible cost that we would expect to pay, namely \$39.30.

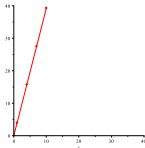
REPRESENTING FUNCTIONS

We might represent the function of the previous example

- numerically

gallons	Cost
1	\$ 3.93
4	\$15.72
7	\$ 27.51
10	\$ 39.30

- graphically



- algebraically

$$C(g) = 3.93g$$

DETERMINING FUNCTION OUTPUT

Determining the output of a function is dependent on the presentation of the function.

- Numerical (-i.e. chart or table): Read off the output.
- Graphical: Draw a vertical line which intersects the x -axis at the input value. Draw a horizontal line which meets the graph at the same place as the vertical line you just drew. The point of intersection of the horizontal line with the y -axis is the function output.
- Algebraic: If we have an equation relating the input variable to the output variable, we simply substitute the input for the input variable and calculate the value of the output variable.

DEFINITION

The process of translating a real-world problem into a usable function is called mathematical modeling. The resulting function along with a description of all input and output variables (including proper units of measure) is referred to as a mathematical model.

NOTE

Mathematical models are often represented by input/output diagrams.

FINDING AND INTERPRETING MODEL OUTPUT

In order to understand a mathematical model, it is important to identify and understand the units of measure involved.

EXAMPLE

The value of a certain piece of property between the years 1980 and 2007 is given by the model

$$v(t) = 2.7(1.083)^t \text{ thousands of dollars.}$$

where t is the number of years since the end of 1980.

- 1 Give the description and the unit of measure for both the input and output variables.
- 2 Draw an input/output diagram for v .
- 3 Graph the value of the property from 1980 to 2007 (-i.e. $0 \leq t \leq 27$).
- 4 What was the land value at the end of 2002?

INTERPRETING MODEL INPUT

In order to find the value of the input variable x which will produce a certain output $f(x)$, we substitute the desired value for $f(x)$ in the algebraic description of our model and solve for the input variable x .

EXAMPLE

Suppose again that the value of a certain piece of property between the years 1980 and 2007 is given by the model

$$v(t) = 2.7(1.083)^t \text{ thousands of dollars.}$$

where t is the number of years since the end of 1980.

- 1 In what year was the land worth \$ 18,000?
- 2 Write a sentence interpreting the result above.

IS IT A FUNCTION?

Recall that a function assigns to each input value a unique (unambiguous) output value.

VERBALLY Ask the question, “Can any specific input value correspond to more than one output value?”

NUMERICALLY Does each input produce only one output? (Does each input value appear only once in the table?)

GRAPHICALLY **Vertical Line Test:** Suppose that we have a graph of data which are related in some way. If we can draw a vertical line which intersects the graph in 2 or more places then the variable whose values are plotted along the y-axis **cannot** be expressed as a function of the variable whose values are plotted along the x-axis.

NOTE

Not all equations are functions. For example $x^2 + y^2 = 1$ is an equation but neither x nor y can be expressed as a function of the other.

If we are given the equation $y^2 - x = 2$, we can express x as a function of y but not the other way around.

NOTE (VERTICAL LINE TEST)

Suppose that we have a graph of data which are related in some way. If we can draw a vertical line which intersects the graph in 2 or more places then the variable whose values are plotted along the y-axis **cannot** be expressed as a function of the variable whose values are plotted along the x-axis.