MTHSC 102 Section 1.1 – Functions: Four Representations

Kevin James

Note

We will typically represent related data in four ways or from four viewpoints.

- Numerically (using a chart or table of data)
- Graphically (using a scatter plot or continuous graph)
- Verbally (using a word description)
- Algebraically (using a mathematical model).

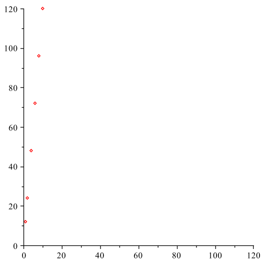
Example

Suppose that we wish to fill in a low swampy area with 120 cubic yards of dirt. Suppose that we will use a truck which is capable of moving 12 cubic yards of soil per load and that we can deliver 1 load of soil per hour around the clock until the job is complete. The amount of soil moved after t hours is given by the following chart.

Elapsed time (hours)	Amount of Soil
1	12
2	24
4	48
6	72
8	96
10	120

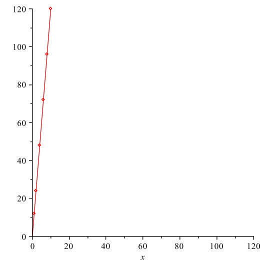
Example continued...

We could also represent the above data by the following plot



Example continued...

Or the following continuous graph



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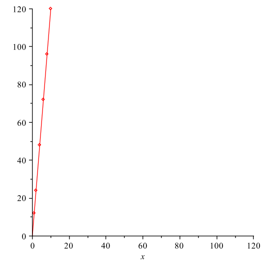

An algebraic description of the amount A of soil moved after t hours is given by

$$A(t) = 12t$$

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Example

If the price of gas is fixed at \$3.93 per gallon and g is the number of gallons that we wish to purchase, then the cost C will be a function of g, because for a given value of g, say $g = 10$ gallons there is only one possible cost that we would expect to pay, namely \$39.30.

We might represent the function of the previous example

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gallons	Cost
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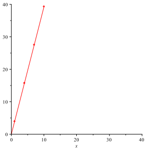

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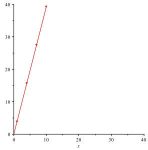

- algebraically

$$C(g) = 3.93g$$

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Determining Function Output

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- Graphical: Draw a vertical line which intersects the x-axis at the input value. Draw a horizontal line which meets the graph at the same place as the vertical line you just drew. The point of intersection of the horizontal line with the y-axis is the function output.

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- Algebraic: If we have an equation relating the input variable to the output variable, we simply substitute the input for the input variable and calculate the value of the output variable.

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Note

Mathematical models are often represented by input/output diagrams.

Finding and Interpreting Model Output

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$$v(t) = 2.7(1.083)^t \text{ thousands of dollars.}$$

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1. Give the description and the unit of measure for both the input and output variables.
2. Draw an input/output diagram for v.
3. Graph the value of the property from 1980 to 2007 (-i.e. $0 \leq t \leq 27$).
4. What was the land value at the end of 2002?

In order to find the value of the input variable x which will produce a certain output $f(x)$, we substitute the desired value for $f(x)$ in the algebraic description of our model and solve for the input variable x.

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EXAMPLE

Suppose again that the value of a certain piece of property between the years 1980 and 2007 is given by the model

$$v(t) = 2.7(1.083)^t \text{ thousands of dollars.}$$

where t is the number of years since the end of 1980.

1. In what year was the land worth \$ 18,000?
2. Write a sentence interpreting the result above.

Recall that a function assigns to each input value a unique (unambiguous) output value.

- **Verbally** Ask the question, "Can any specific input value correspond to more than one output value?"
- **Numerically** Does each input produce only one output? (Does each input value appear only once in the table?)
- **Graphically** **Vertical Line Test:** Suppose that we have a graph of data which are related in some way. If we can draw a vertical line which intersects the graph in 2 or more places then the variable whose values are plotted along the y-axis **cannot** be expressed as a function of the variable whose values are plotted along the x-axis.

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Not all equations are functions. For example $x^2 + y^2 = 1$ is an equation but neither x nor y can be expressed as a function of the other.

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If we are given the equation $y^2 - x = 2$, we can express x as a function of y but not the other way around.

Note (Vertical Line Test)

Suppose that we have a graph of data which are related in some way. If we can draw a vertical line which intersects the graph in 2 or more places then the variable whose values are plotted along the y-axis **cannot** be expressed as a function of the variable whose values are plotted along the x-axis.