MTHSC 102 Section 1.2 – Function Behavior and End Behavior Limits
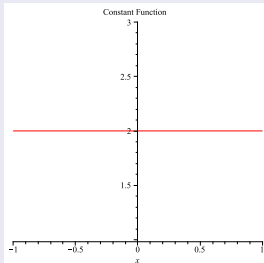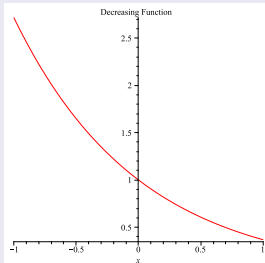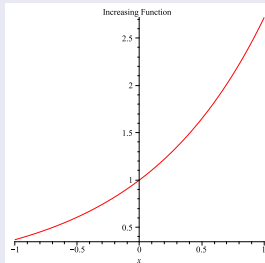
Kevin James

DEFINITION

A function f defined over an input interval is said to be

INCREASING if the output values increase as the input values increase.

DECREASING if the output values decrease as the input values increase.

CONSTANT if the output values remain the same as the input values increase.
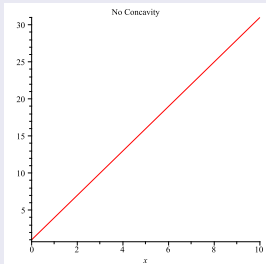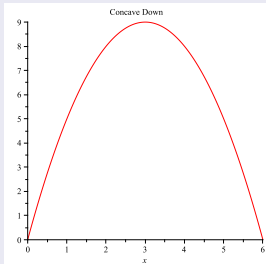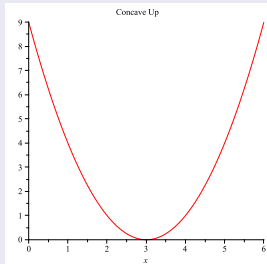
EXAMPLE

DEFINITION

A function f defined over an input interval is said to be

CONCAVE UP if the graph of the function appears to be a portion of an arc opening upwards.

CONCAVE DOWN if the graph of the function appears to be a portion of an arc opening downwards.
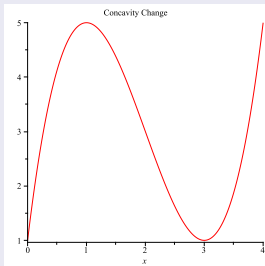
EXAMPLE

DEFINITION

A point on the graph of a continuous function where the concavity changes is called an <u>inflection point</u>.
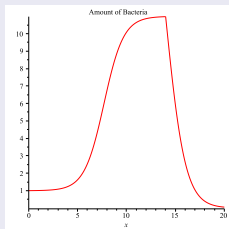
EXAMPLE

The graph of $f(x) = x^3 - 6x^2 + 9x + 1$

has an inflection point at $(2, 3)$.

EXAMPLE

The amount A of a certain bacteria present in a host t days after initial infection for the first 20 days is given by the following graph.

Amount of Bacteria

1. Identify intervals on which A is increasing, decreasing or constant.

2. Identify intervals on which A is concave up, down or neither.

3. Using the information above, describe what is happening to the bacteria population in the host.
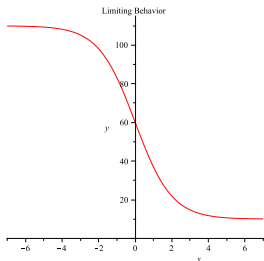
DEFINITION

The term <u>end behavior</u> refers to the behavior of the output values of a function as the input values either increase without bound or decrease without bound.
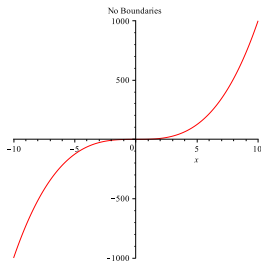
NOTE

There are 3 possible types of end behavior of a function

- The output values may approach or equal a certain number.
- The output values may increase or decrease without bound.
- The output values may oscillate and fail to approach a particular number.
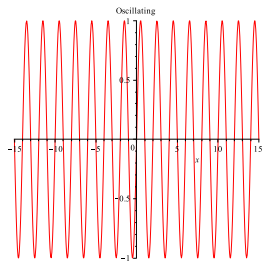
$$\lim_{x \to \infty} f(x) = 10 \qquad \lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to \infty} f(x) = \text{DNE}$$

$$\lim_{x \to -\infty} f(x) = 110 \qquad \lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to \infty} f(x) = \text{DNE}$$

DEFINITION

We write

1. $\lim_{x \to \infty} f(x) = \infty$ if f increases without bound as x gets large and positive.

2. $\lim_{x \to \infty} f(x) = -\infty$ if f decreases without bound as x gets large and positive.

3. $\lim_{x \to \infty} f(x) = L$ if f approaches L as x gets large and positive. Note that this means that f has a horizontal asymptote at $y = L$.

4. $\lim_{x \to -\infty} f(x) = \infty$ if f increases without bound as x gets large and negative.

5. $\lim_{x \to -\infty} f(x) = -\infty$ if f decreases without bound as x gets large and negative.

6. $\lim_{x \to -\infty} f(x) = L$ if f approaches L as x gets large and negative. Note that this means that f has a horizontal asymptote at $y = L$.

1 $\lim_{x \to \infty} 3x + 1 = \underline{\hspace{2cm}}$

2 $\lim_{x \to -\infty} 3x + 1 = \underline{\hspace{2cm}}$

3 $\lim_{x \to \infty} e^x = \underline{\hspace{2cm}}$

4 $\lim_{x \to -\infty} 3^x = \underline{\hspace{2cm}}$

5 $\lim_{x \to \infty} \frac{10}{1 + e^{-x}} = \underline{\hspace{2cm}}$

6 $\lim_{x \to -\infty} \frac{10}{1 + e^{-x}} = \underline{\hspace{2cm}}$

DEFINITION

If $\lim_{x \to \infty} f(x) = L_1$ and/or $\lim_{x \to -\infty} f(x) = L_2$ then we say that the line(s) $y = L_1$ and/or $y = L_2$ are horizontal asymptotes.