MTHSC 102 Section 1.2 – Function Behavior and End Behavior Limits

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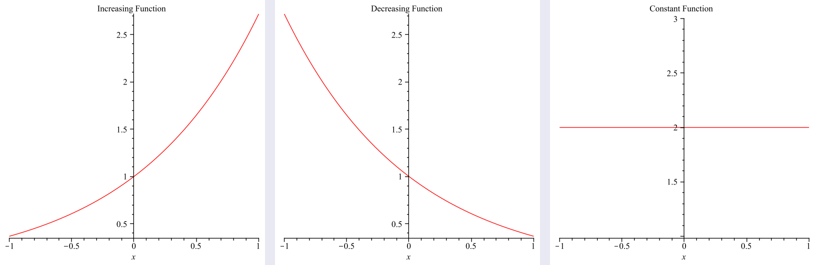
Definition

A function f defined over an input interval is said to be

- **Increasing** if the output values increase as the input values increase.
- **Decreasing** if the output values decrease as the input values increase.
- **Constant** if the output values remain the same as the input values increase.

Example



Increasing Function
Decreasing Function
Constant Function
x

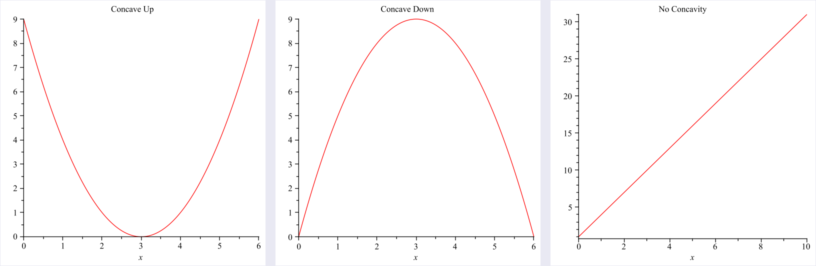
Definition

A function f defined over an input interval is said to be

Concave Up if the graph of the function appears to be a portion of an arc opening upwards.

Concave Down if the graph of the function appears to be a portion of an arc opening downwards.

Example



Definition

A point on the graph of a continuous function where the concavity changes is called an inflection point.

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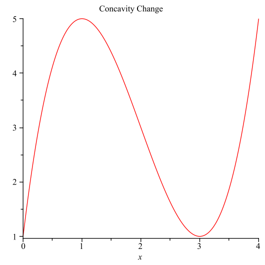
Example

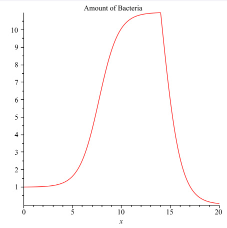
The graph of $f(x) = x^3 - 6x^2 + 9x + 1$



has an inflection point at $(2, 3)$.

Example

The amount A of a certain bacteria present in a host t days after initial infection for the first 20 days is given by the following graph.



1. Identify intervals on which A is increasing, decreasing or constant.
2. Identify intervals on which A is concave up, down or neither.
3. Using the information above, describe what is happening to the bacteria population in the host.

Definition

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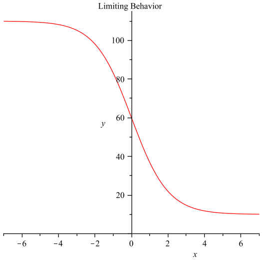
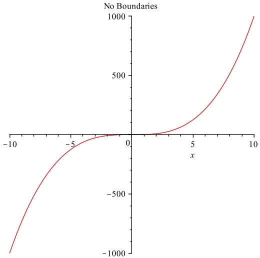
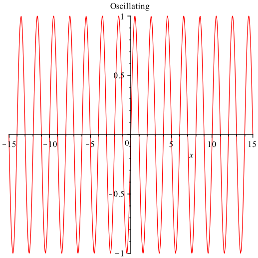
NOTE

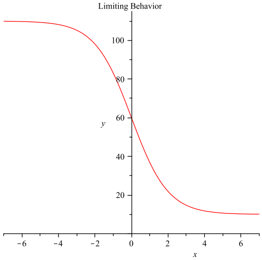
There are 3 possible types of end behavior of a function

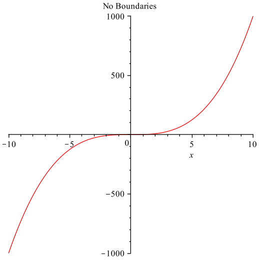
- The output values may approach or equal a certain number.
- The output values may increase or decrease without bound.
- The output values may oscillate and fail to approach a particular number.

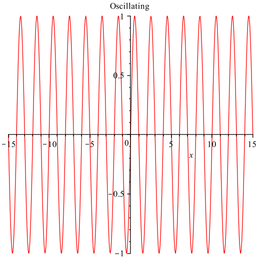

Limiting Behavior
100
80
60
y
40
20
−6
−4
−2
0
2
4
6
x


No Boundaries
1000
500
−10
−5
0
5
10
x
−500
−1000


Oscillating
1
0.5
−15
−10
−5
0
5
10
15
x
−0.5
−1







$\lim_{x \to \infty} f(x) = 10$ $\quad$ $\lim_{x \to \infty} f(x) = \infty$ $\quad$ $\lim_{x \to \infty} f(x) = \text{DNE}$

$\lim_{x \to -\infty} f(x) = 110$ $\quad$ $\lim_{x \to -\infty} f(x) = -\infty$ $\quad$ $\lim_{x \to \infty} f(x) = \text{DNE}$

Definition

We write

1. $\lim_{x\to\infty} f(x) = \infty$ if f increases without bound as x gets large and positive.
2. $\lim_{x\to\infty} f(x) = -\infty$ if f decreases without bound as x gets large and positive.
3. $\lim_{x\to\infty} f(x) = L$ if f approaches L as x gets large and positive. Note that this means that f has a horizontal asymptote at $y = L$.
4. $\lim_{x\to-\infty} f(x) = \infty$ if f increases without bound as x gets large and negative.
5. $\lim_{x\to-\infty} f(x) = -\infty$ if f decreases without bound as x gets large and negative.
6. $\lim_{x\to-\infty} f(x) = L$ if f approaches L as x gets large and negative. Note that this means that f has a horizontal asymptote at $y = L$.

Example

1. $\lim_{x\to\infty} 3x + 1 =$ __________
2. $\lim_{x\to-\infty} 3x + 1 =$ __________
3. $\lim_{x\to\infty} e^x =$ __________
4. $\lim_{x\to-\infty} 3^x =$ __________
5. $\lim_{x\to\infty} \frac{10}{1+e^{-x}} =$ __________
6. $\lim_{x\to-\infty} \frac{10}{1+e^{-x}} =$ __________

Example

1. $\lim_{x\to\infty} 3x+1 =$ ____________
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6. $\lim_{x\to-\infty} \frac{10}{1+e^{-x}} =$ ____________

Definition

If $\lim_{x\to\infty} f(x) = L_1$ and/or $\lim_{x\to-\infty} f(x) = L_2$ then we say that the line(s) $y = L_1$ and/or $y = L_2$ are <u>horizontal asymptotes</u>.