MTHSC 102 Section 1.3 – Limits and Continuity

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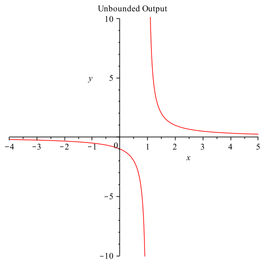
Example

Consider the function whose graph is given below.



1. Discuss the end behavior of the function. Are there horizontal asymptotes?
2. Discuss the behavior of the function near $x = 1$.

Definition

Suppose that f is a function defined on some interval containing c (except f may not be defined at c itself).

1. If $f(x)$ becomes very near a number L_ℓ as x becomes very near and to the left of c then we write

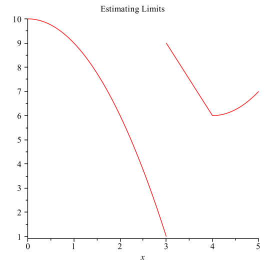
$$\lim_{x \to c^-} f(x) = L_\ell.$$

2. If $f(x)$ becomes very near a number L_r as x becomes very near and to the right of c then we write

$$\lim_{x \to c^+} f(x) = L_\ell.$$

Example

Consider the function f whose graph is the following.



1. Estimate $\lim_{x \to 3^-} f(x)$.
2. Estimate $\lim_{x \to 3^+} f(x)$.
3. Estimate $\lim_{x \to 4^-} f(x)$.
4. Estimate $\lim_{x \to 4^+} f(x)$.

Definition

Suppose that f is a function defined on some interval containing c (except f may not be defined at c itself). If it is true that

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L,$$

then we say that the <u>limit of f as x approaches c is L</u> and write

$$\lim_{x \to c} f(x) = L$$

Example

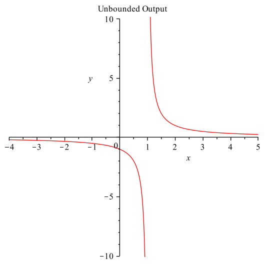
What can you say about the limits near 3 and 4 in the previous example?

Definition

If $\lim_{x\to c^-} = \pm\infty$ and $\lim_{x\to c^+} = \pm\infty$ then we say that the line $x = c$ is a vertical asymptote for the function $f(x)$.

Example

The function



has a vertical asymptote at $x = 1$.

Example

Consider the function $f(x) = \frac{x^2-9}{x-3}$.

1. Compute $\lim_{x\to 3^-} f(x)$ numerically.
2. Compute $\lim_{x\to 3^+} f(x)$ numerically.
3. Does the $\lim_{x\to 3} f(x)$ exist?
4. Does $f(3)$ exist?

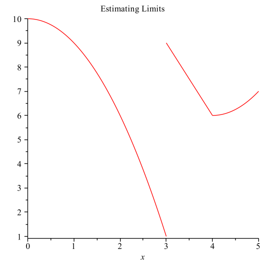
Definition

A function f is continuous at c if

1. $f(c)$ exists.
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

Example

The function $f(x)$ from before whose graph is below



is continuous at 4 but not at 3. It is also continuous at 2.

Definition

A function f is continuous on an open interval if it is continuous at every point in the interval.
A function f is continuous everywhere if it is continuous at all points.
We call functions which are continuous everywhere continuous functions.

Example

The function from the previous example is continuous on the intervals $(0, 3)$ and $(3, 5)$.