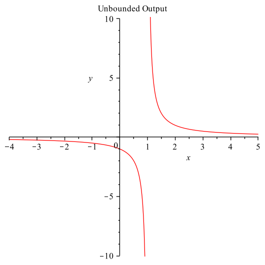
MTHSC 102 Section 1.3 – Limits and Continuity

Kevin James

Example

Consider the function whose graph is given below.



1. Discuss the end behavior of the function. Are there horizontal asymptotes?
2. Discuss the behavior of the function near $x = 1$.

Definition

Suppose that f is a function defined on some interval containing c (except f may not be defined at c itself).

1. If $f(x)$ becomes very near a number L_ℓ as x becomes very near and to the left of c then we write

$$\lim_{x \to c^-} f(x) = L_\ell.$$

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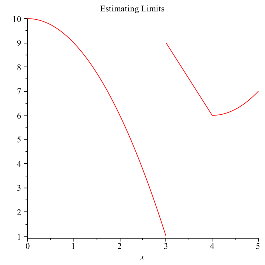
2. If $f(x)$ becomes very near a number L_r as x becomes very near and to the right of c then we write

$$\lim_{x \to c^+} f(x) = L_\ell.$$

Example

Consider the function f whose graph is the following.



1. Estimate $\lim_{x\to 3^-} f(x)$.
2. Estimate $\lim_{x\to 3^+} f(x)$.
3. Estimate $\lim_{x\to 4^-} f(x)$.
4. Estimate $\lim_{x\to 4^+} f(x)$.

Definition

Suppose that f is a function defined on some interval containing c (except f may not be defined at c itself). If it is true that

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L,$$

then we say that the <u>limit of f as x approaches c is L</u> and write

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Example

What can you say about the limits near 3 and 4 in the previous example?

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If $\lim_{x \to c^-} = \pm\infty$ and $\lim_{x \to c^+} = \pm\infty$ then we say that the line $x = c$ is a vertical asymptote for the function $f(x)$.

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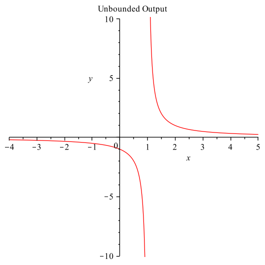
Example

The function



has a vertical asymptote at $x = 1$.

Example

Consider the function $f(x) = \frac{x^2-9}{x-3}$.

1. Compute $\lim_{x\to 3^-} f(x)$ numerically.
2. Compute $\lim_{x\to 3^+} f(x)$ numerically.
3. Does the $\lim_{x\to 3} f(x)$ exist?
4. Does $f(3)$ exist?

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Consider the function $f(x) = \frac{x^2-9}{x-3}$.

1. Compute $\lim_{x \to 3^-} f(x)$ numerically.
2. Compute $\lim_{x \to 3^+} f(x)$ numerically.
3. Does the $\lim_{x \to 3} f(x)$ exist?
4. Does $f(3)$ exist?

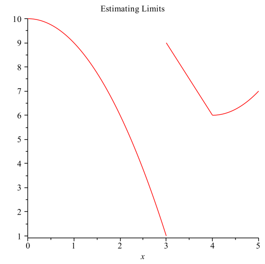
Definition

A function f is <u>continuous at c</u> if

1. $f(c)$ exists.
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Example

The function $f(x)$ from before whose graph is below



is continuous at 4 but not at 3. It is also continuous at 2.

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Example

The function from the previous example is continuous on the intervals $(0, 3)$ and $(3, 5)$.