

MTHSC 102 SECTION 1.4 – LINEAR FUNCTIONS AND MODELS

Kevin James

LINEAR MODEL

ALGEBRAIC A linear model has an equation of the form
 $f(x) = ax + b$ where a and b are numbers.

GRAPHICALLY The graph of a linear model is a straight line.

VERBALLY A linear model has a constant rate of change.

Consider a newspaper delivery team that makes weekly deliveries of newspapers and devotes their Saturday mornings to selling new subscriptions. Suppose that the number of customers during week w is given by the following table where w is the number of weeks since the beginning of business.

Weeks	Number of customers
0	70
1	77
2	84
3	91
4	98
5	105

Graph the above data.

Give a linear model describing this data.

FACT

All linear functions have the form $f(x) = ax + b$ where a and b are constants.

a is called the slope (which measures the line's steepness), and b is the vertical axis intercept.

FACT

Given any two data points for a linear function

<i>input</i>	<i>output</i>
x_1	y_1
x_2	y_2

we can compute the slope as follows

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}.$$

EXAMPLE

Compute the slope of the linear function from the previous example.

NOTE

The slope of a graph at a particular point is a measure of how quickly the graph is changing at that point. This measure is called the rate of change of the function at that point.

SLOPE AND END BEHAVIOR OF LINEAR FUNCTIONS

NOTE

Suppose that $f(x)$ is a linear function.

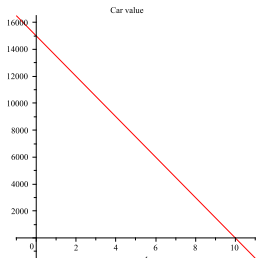
- 1 If the slope of f is positive, then $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow -\infty} f(x) = -\infty$
- 2 If the slope of f is negative, then $\lim_{x \rightarrow \infty} f(x) = -\infty$ and $\lim_{x \rightarrow -\infty} f(x) = \infty$
- 3 If the slope of f is 0, then we have $f(x) = b$ where b is a number. In this case, $\lim_{x \rightarrow \infty} f(x) = b$ and $\lim_{x \rightarrow -\infty} f(x) = b$
- 4 The slope measures the relative steepness of the graph of a linear function.
- 5 A vertical line has an undefined slope.

NOTE

Make sure that all of your models have the following crucial elements.

- 1 An equation
- 2 A description including units of measure of the output variable.
- 3 A description of the input variable including its units of measure
- 4 When possible, the interval of input data used to find the model.

Suppose that the resale value of a used car is represented graphically by the following graph.



- 1 Locate the point where the input is 0 and the point where the output is 0.
- 2 Calculate the slope of the graph.
- 3 Does the slope depend on the direction in which it is calculated?
- 4 Interpret the slope.
- 5 Write a linear model for the graph.

DEFINITION

Suppose we are considering the graph of a linear model or function.

- 1 If the graph of the function rises as we move to the right (-i.e. if the slope is positive) then we say that the function is increasing.
- 2 If the graph of the function falls as we move to the right (-i.e. if the slope is negative) then we say that the function is decreasing.
- 3 If the function is neither increasing nor decreasing (-i.e. if the slope is 0) then we say that the function is constant.

FINDING A LINEAR MODEL FROM DATA

Suppose that the taxes paid by a certain business during the years from 1999 to 2004 are given by the following table.

Year	1999	2000	2001	2002	2003	2004
Tax	\$ 2503	\$ 3008	\$ 3513	\$ 4018	\$ 4523	\$ 5028

Consider the first differences (-i.e. the differences in output corresponding to successive equally spaced inputs),

Tax	\$ 2503		\$ 3008		\$ 3513		\$ 4018		\$ 4523		\$ 5028
Diff.		\$505		\$505		\$505		\$505		\$505	

we see that they are all the same, namely \$505.

Thus the data can be modeled by a linear function.

The slope is 505 dollars per year.

So letting t denote years past 1999, we have an intercept of \$2503 and the linear model is

$$T(t) = 505t + 2503 \text{ dollars,}$$

where T denotes the amount of taxes paid in the year t years after 1999.

NOTE

Our choice to let t denote years past 1999 was arbitrary. This is called aligning our data. A good choice of data alignment will often result in smaller constants in our models. For example, if the model is linear, alignment will not change the slope but will change the vertical axis intercept.

DEFINITION

We typically develop a model based on data in a certain range. If we use our model to predict the output for an input lying within this range this process is called interpolation. If we use our model to predict the output for an input lying outside this range, this process is called extrapolation. Predictions involving extrapolation should be viewed with skepticism and caution because we may not have any reason to believe that the real-world process which we have modeled will continue to behave in the same way outside of the range that we have observed.



APPROXIMATELY LINEAR DATA

In the previous example, the data formed a straight line.

In many real-world applications, we may have data which nearly lie on a line.

For example, if the table of data in the previous example had been

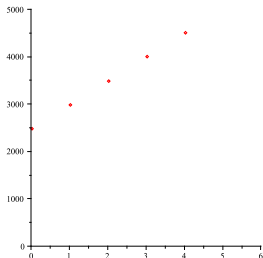
Year	1999	2000	2001	2002	2003	2004
Tax	\$ 2503	\$ 3008	\$ 3513	\$ 4021	\$ 4526	\$ 5031

Calculating the first differences we have

Tax	\$2502		\$3008		\$3512		\$4019		\$4524		\$5030
Diff.		\$506		\$504		\$507		\$505		\$506	

Thus our data is not linear, but is nearly linear.

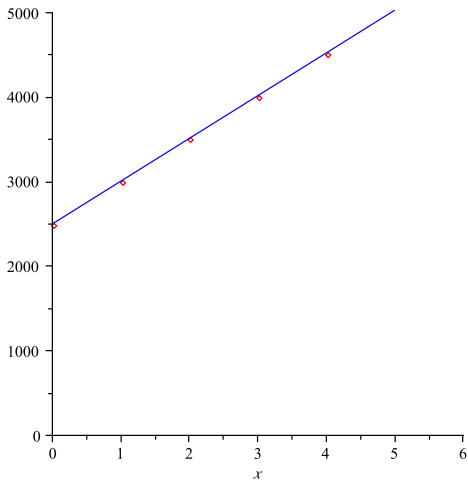
Here is a scatter plot of the data from the previous page.



We can use our calculators to find the line that is the best fit to the data, namely

$$T(t) = 505.7714286t + 2502.571429$$

Let's graph the line of best fit over our scatter plot.



RULES TO ENSURE ACCURACY IN COMPUTATIONS

ROUNDING Round results in a way that makes sense in the context of the problem. (-e.g. Do not report more than 2 decimal places if the unit is dollars).

ROUNDING Round results to the same accuracy as the least accurate of your output data.

UNITS A number without proper labeling with units is useless.

MODELS Always use unrounded models for computations.

COMPUTATION Postpone rounding until the final result is computed.