MTHSC 102 Section 1.5 – Exponential Functions and Models

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Definition

Algebraically An exponential function has an equation of the form

$$f(x) = ab^x.$$

The constant a is called the starting value. The constant b is called the constant multiplier.

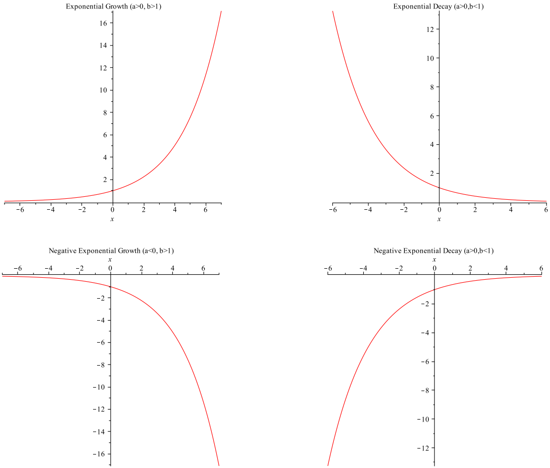
Verbally An exponential has a constant percentage change. The percentage change is $(b - 1) \times 100\%$. Alternatively, if p is the percent change then the constant multiplier is $b = [(p/100) + 1]\%$.

Example

$f(x) = 300(1.03)^x$ is an exponential function.
The initial (or starting value) is $f(0) = 300$.
The constant multiplier is 1.03.
The constant percent change is 3%.

Graphs of Exponential Functions

There are four possibilities for the graphs of these functions. The first two are the most common in applications.



Note

The concavity of an exponential function is determined by the starting value a. If $a > 0$ then the function is concave up. If $a < 0$ then the function is concave down.

Note

The end behavior is determined by the constant multiplier b. Suppose that $f(x) = ab^x$ is an exponential function.

$0 < b < 1$ $\lim_{x\to\infty} f(x) = 0.$

$b > 1$ $\lim_{x\to\infty} f(x) = \begin{cases} \infty & \text{if } a > 0, \\ -\infty & \text{if } a < 0. \end{cases}$

Example

During 2001 iPod sales were 0.14 million units and until 2005 sales increased approximately 260% each year.

1. Why is an exponential model appropriate for iPod sales?
2. Find a model for iPod sales.
3. According to the model what were the 2006 iPod sales?

Definition

For any function $f(x)$, the percentage change between two data points is a measure of the relative change between two output values. Percentage change in output as input changes from x_1 to x_2 is calculated as

$$\text{percentage change} = \frac{f(x_2) - f(x_1)}{f(x_1)} \times 100\%.$$

Note

Exponential functions exhibit a constant percentage change.

Example (Modeling from Data)

The following data represents the dwindling population in a mill town t years after the closing of the mill.

Year	0	1	2	3	4	5	6
Population	7290	5978	4902	4020	3296	2703	2216

Consider the first differences

Pop.	7290	5978	4902	4020	3296	2703	2216
Diff.		-1312	-1076	-882	-724	-593	-487
%		-17.997	-17.999	-17.993	-18.010	-17.992	-18.017

Since the percentage differences are nearly constant, the use of an exponential model is appropriate.

Using our calculators we have the model

$P(t) = 7290.366(0.819995)^t$ people.

Note

A good choice of alignment of input data may produce simpler models. Graphically, alignment of the input data simply translates the graph horizontally.