MTHSC 102 Section 1.5 – Exponential Functions and Models

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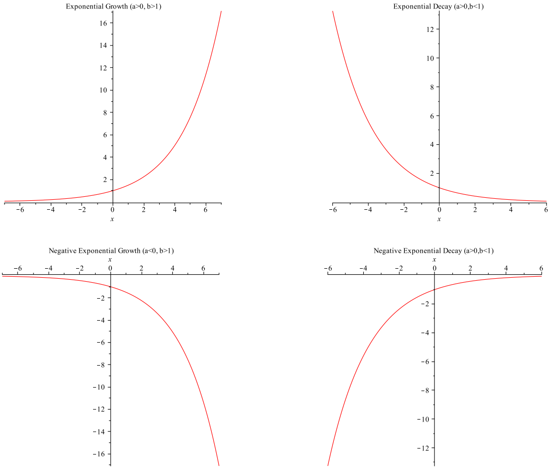
The constant multiplier is 1.03.
The constant percent change is 3%.

There are four possibilities for the graphs of these functions. The first two are the most common in applications.



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$$b > 1 \quad \lim_{x\to\infty} f(x) = \begin{cases} \infty & \text{if } a > 0, \\ -\infty & \text{if } a < 0. \end{cases}$$

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2. Find a model for iPod sales.
3. According to the model what were the 2006 iPod sales?

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For any function $f(x)$, the percentage change between two data points is a measure of the relative change between two output values. Percentage change in output as input changes from x_1 to x_2 is calculated as

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Note

Exponential functions exhibit a constant percentage change.

Example (Modeling from Data)

The following data represents the dwindling population in a mill town t years after the closing of the mill.

Year	0	1	2	3	4	5	6
Population	7290	5978	4902	4020	3296	2703	2216

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Using our calculators we have the model
$P(t) = 7290.366(0.819995)^t$ people.

Note

A good choice of alignment of input data may produce simpler models. Graphically, alignment of the input data simply translates the graph horizontally.