MTHSC 102 Section 1.6 – Models in Finance

Kevin James

Present and Future Value and Interest

Definition

1. The value of an investment (or loan) at time $t = 0$ is called the present value.
   The present value of an investment is also called the principal.
   The present value of a loan is also called the face value.
2. The value of an investment (or loan) at some time $t > 0$ in the future is called the future value.
3. Interest is the increase between the present value and the future value.
4. The decimal form of the Annual Percentage Rate (APR) is the nominal rate which is usually denoted r.

$$APR = 100r\%.$$

Definition (Simple Interest)

The accumulated interest i after t years at an annual interest rate r on a present value of P dollars is calculated as

$$I(t) = Prt \quad \text{dollars.}$$

The future value at time t is give by

$$F_s(t) = P(1 + rt) \quad \text{dollars.}$$

Example (Simple Interest)

Suppose that a worker invests \$ 1,000 for five years at a simple interest rate of 4% annually.

The present value of the investment is $P = \$\ 1,000$.

The interest on the investment is \$ 40 per year or \$ 200 after 5 years.

The future value of the investment after 5 years is $F(5) = \$1,200$.

Definition (Compound Interest)

The future value at time t in years of a loan (or investment) with a present value of P dollars is calculated as

$$F_c(t) = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + i\right)^{nt} \quad \text{dollars,}$$

where n is the number of compoundings per year, nt is the total number of compoundings during the term of t years, r is the nominal rate, and $i = \frac{r}{n}$ is the interest rate per compounding period.

Example

An investment of \$ 1,000 with an APR of 2.1% and quarterly compounding would grow to a future value of

$$1000\left(1 + \frac{0.021}{4}\right)^{4\times 5} = \$\ 1110.41$$

in five years.

Discounting an Investment

This refers to the problem of determining the present value (-i.e. the amount to be invested today) of a certain type of investment for some specified number of years in order to achieve a specified future value.

Example

Determine the amount that must be invested today at 12% APR compounded quarterly to obtain \$ 1,000,000 payable in 40 years.

Solution

The future value function of such an investment with principal P after $t = 30$ years is

$$F(30) = P\left(1 + \frac{0.12}{4}\right)^{4\times 30} = P(1.03)^{120} \approx P(34.710987)$$

So, $P = \$\ 28,809.33$ would be sufficient.

DEFINITION

1. The annual percentage yield (APY) of an investment is the interest that would be necessary to obtain the same amount per year with yearly compounding. Computing the APY for various types of investments gives a way to compare them easily.
2. The decimal form of the APY is the effective rate of interest.

NOTE

The effective rate for an investment (or loan) with a nominal rate of r compounded n times per year is given by

$$\left(1+\frac{r}{n}\right)^n - 1.$$

So the APY is $100 \times \left(\left(1+\frac{r}{n}\right)^n - 1\right)\%$

Example

In the previous example we considered an investment at 12% APR compounded quarterly.

The effective rate of this investment is

$$\left(1 + \frac{0.12}{4}\right)^4 - 1 = 1.03^4 - 1 \approx 0.1255088.$$

So the APY is approximately 12.55%.

Continuously Compounded Interest Formula

The amount accumulated in an account after t years when P dollars are invested at an (APR) of $100r\%$ compounded continuously is

$$F_c(t) = Pe^{rt} \text{ dollars.}$$

Note

The effective rate of an investment with nominal rate r and continuous compounding is $e^r - 1$.
Thus the APY is $100 \times (e^r - 1)\,\%$.

Doubling Time of an Investment

In order to find the time necessary for a given investment with future value $F(t)$ to double, we solve the equation

$$2P = F(t),$$

where P is the present value (or principal).

Note

1. For investments with a nominal rate of r compounded n times per year, we need to solve
$$2P = P\left(1+\frac{r}{n}\right)^{nt} \Leftrightarrow 2 = \left(1+\frac{r}{n}\right)^{nt}$$
for t.
2. For investments with a nominal rate of r continuously compounded, we need to solve
$$2P = Pe^{rt} \Leftrightarrow 2 = e^{rt}$$
for t.