MTHSC 102 Section 1.6 – Models in Finance

Kevin James

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Present and Future Value and Interest

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Definition (Simple Interest)

The accumulated interest i after t years at an annual interest rate r on a present value of P dollars is calculated as

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The future value at time t is give by

$$F_s(t) = P(1 + rt) \quad \text{dollars.}$$

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The future value of the investment after 5 years is $F(5) = \$1,200$.

Definition (Compound Interest)

The future value at time t in years of a loan (or investment) with a present value of P dollars is calculated as

$$F_c(t) = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + i\right)^{nt} \quad \text{dollars,}$$

where n is the number of compoundings per year, nt is the total number of compoundings during the term of t years, r is the nominal rate, and $i = \frac{r}{n}$ is the interest rate per compounding period.

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Example

An investment of \$ 1,000 with an APR of 2.1% and quarterly compounding would grow to a future value of

$$1000\left(1 + \frac{0.021}{4}\right)^{4\times 5} = \$\ 1110.41$$

in five years.

This refers to the problem of determining the present value (-i.e. the amount to be invested today) of a certain type of investment for some specified number of years in order to achieve a specified future value.

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Determine the amount that must be invested today at 12% APR compounded quarterly to obtain $ 1,000,000 payable in 40 years.

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SOLUTION

The future value function of such an investment with principal P after $t = 30$ years is

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So, $P = \$\ 28,809.33$ would be sufficient.

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Comparing Investments APR vs APY

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Note

The effective rate for an investment (or loan) with a nominal rate of r compounded n times per year is given by

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So the APY is approximately 12.55%.

Continuously Compounded Interest Formula

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The effective rate of an investment with nominal rate r and continuous compounding is $e^r - 1$.
Thus the APY is $100 \times (e^r - 1)\,\%$.

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$$2P = F(t),$$

where P is the present value (or principal).

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1. For investments with a nominal rate of r compounded n times per year, we need to solve
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