MTHSC 102 Section 1.7 – Constructed Functions

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Before describing combination of functions and the meaning of such combinations, here are some typical variables which arise in business and their relationships.

Business Term	Description
Fixed Costs	Costs that occur regardless of the number of items produces
Variable Costs	Productions costs
Total Costs	Fixed Cost + Variable Costs
Average Cost	$\frac{\text{total cost}}{\#\text{ items produced}}$
Revenue	Amount of income the business recieves
	Revenue = Profit + Total Cost
	Revenue = Price $\times$ Demand
Cost	Same as total cost
Profit	Profit = Revenue - Cost
Break-even point	Point where Cost = Revenue

Note

Functions can be constructed using addition and subtraction if

- The inputs of the two functions have the same description and units.
- The outputs of the two functions have the same units AND if there is a way to describe the resulting output if the functions are added or subtracted.
- In many examples the given functions may fail one of the above criteria, but can be adjusted so that the criteria are satisfied.

Example

Consider the total cost of a dairy business during a certain month. The fixed costs for the dairy for that month are \$ 21,000 and the variable cost incurred on day d of the month in question is given by $v(d) = -0.3d^2 + 6d + 250$ where $0 \leq d \leq 30$. Express the total cost of operations on day d of the month.

Multiplication and Division of functions

Functions can be constructed using multiplication or division if

- The inputs for both functions have the same description and units.
- The output units for the 2 functions are well defined when they are multiplied or divided.
- There is a meaningful description of the resulting output when the functions have been multiplied or divided.

Example

Again consider the dairy business from before. Suppose that the price of milk on day d is given by $M(d) = 0.019d + 1.85$ dollars per gallon $(0 \leq d \leq 30)$ and that $S(d) = 1.7 + 0.4(0.82^d)$ thousand gallons of milk were sold on day d.

Draw separate input/output diagrams for M and S.

Determine if M and S are compatible for multiplication.

What is the resulting output unit of measure for $(M \cdot S)(d)$.

Draw an input/output diagram for the multiplication function.

Write a function for daily revenue from milk sales.

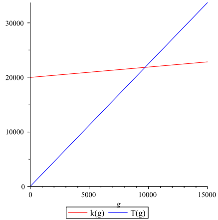
Example

Continuing our example from before, suppose that we know that the total production on day d was $Q(d) = -1.5d^2 + 32d + 1803$ gallons of milk and that the total cost incurred on day d was $C(d) = -0.31d^2 + 6.2d + 1035$ dollars where again $0 \leq d \leq 30$. What is the average cost of production of a gallon of milk on each day?

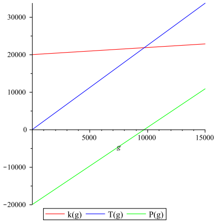
Example

Suppose that the total cost to produce g gallons of milk during a particular month is $K(g) = 20000 + 0.19g$ dollars.
Suppose also that the average price of milk during the month is \$ 2.25. Then the revenue from g gallons of milk can be modeled by $T(g) = 2.25g$ dollars.
Graph $K(g)$ and $T(g)$.
Write a function for the profit for the month if g gallons are produced and graph this function.
How much milk needs to be produced for the dairy to break even for the month?

The following is a graph of the cost and revenue.



The following graph also includes the profit



Definition

Given two functions f and g we can form the composition function $g \circ f(x) = g(f(x))$ if the output values of f can always be used as input values for g. It must be the case also that the unit measure of the output values of f are identical to the input units of g.

Example

Consider the descriptions for the following functions.
$C(p) =$ parts per million of contamination in a lake when the population of the surrounding community is p people.
$p(t) =$ the population in thousands of people of the lakeside community in year t.
Draw an input/output diagram for the composition function that gives the contamination in a lake as a function of time.
Suppose that $C(p) = \sqrt{p}$ parts per million and that $p(t) = 0.4t^2 + 2.5$ thousand people t years after 1980 $(t \geq 0)$.
Write a function that gives the lake contamination as a function of time.

DEFINITION

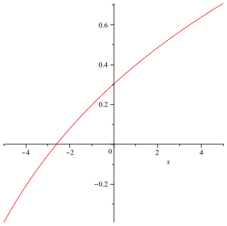
A function is one to one if each of its output values corresponds to precisely one input values. (-i.e. it does not have repeated output values).

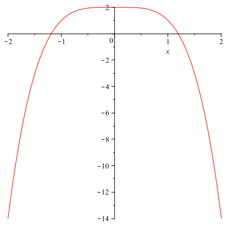
HORIZONTAL LINE TEST

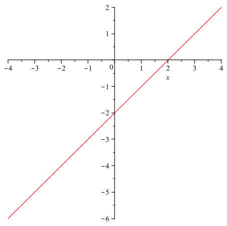
A function is one-to-one if any horizontal line meets the graph of the function in at most one place. If there is a horizontal line meeting the function in more than one place then the function is not one-to-one.

Example

Which of the following are graphs of one-to-one functions?







Definition

For a one-to-one function $g(x)$, the function $f(x)$ is an inverse of g if f sends the output values of g to the corresponding input values. That is,

$$f(g(x)) = x \text{ and } g(f(x)) = x.$$

Example

The underwater pressure measured in atmospheres (atm) d feet below the surface is given numerically by the following table

Depth (ft)	0	33	66	99	132
Pressure (atm)	1	2	3	4	5

This data has the linear model

$$p(d) = \frac{1}{33}d + 1 \text{ atm.}$$

The inverse function which gives the depth d at which a pressure of p atmospheres will be experienced is given numerically by interchanging the rows of the above table.

Pressure (atm)	1	2	3	4	5
Depth (ft)	0	33	66	99	132

This data has the linear model

$$d(p) = 33p - 33 \text{ feet.}$$

Note

- The inverse of a linear function is again linear.
- The inverse of an exponential function is a logarithmic function (see section 1.8).

Composition Property of Inverse Functions

Recall that if f and g are inverse functions then

$$f(g(x)) = (f \circ g)(x) = x,$$

and

$$g(f(x)) = (g \circ f)(x) = x.$$