MTHSC 102 Section 1.7 – Constructed Functions

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Business Term	Description
Fixed Costs	Costs that occur regardless of the number of items produces
Variable Costs	Productions costs
Total Costs	Fixed Cost + Variable Costs
Average Cost	$\frac{\text{total cost}}{\#\text{ items produced}}$
Revenue	Amount of income the business recieves
	Revenue = Profit + Total Cost
	Revenue = Price $\times$ Demand
Cost	Same as total cost
Profit	Profit = Revenue - Cost
Break-even point	Point where Cost = Revenue

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- The outputs of the two functions have the same units AND if there is a way to describe the resulting output if the functions are added or subtracted.
- In many examples the given functions may fail one of the above criteria, but can be adjusted so that the criteria are satisfied.

Example

Consider the total cost of a dairy business during a certain month. The fixed costs for the dairy for that month are \$ 21,000 and the variable cost incurred on day d of the month in question is given by $v(d) = -0.3d^2 + 6d + 250$ where $0 \leq d \leq 30$. Express the total cost of operations on day d of the month.

Multiplication and Division of functions

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- The output units for the 2 functions are well defined when they are multiplied or divided.
- There is a meaningful description of the resulting output when the functions have been multiplied or divided.

Example

Again consider the dairy business from before. Suppose that the price of milk on day d is given by $M(d) = 0.019d + 1.85$ dollars per gallon $(0 \leq d \leq 30)$ and that $S(d) = 1.7 + 0.4(0.82^d)$ thousand gallons of milk were sold on day d.

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Write a function for daily revenue from milk sales.

Example

Continuing our example from before, suppose that we know that the total production on day d was $Q(d) = -1.5d^2 + 32d + 1803$ gallons of milk and that the total cost incurred on day d was $C(d) = -0.31d^2 + 6.2d + 1035$ dollars where again $0 \leq d \leq 30$.

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Graph $K(g)$ and $T(g)$.

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Write a function for the profit for the month if g gallons are produced and graph this function.

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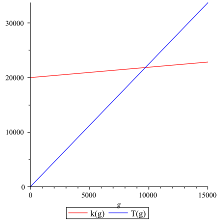
Graph $K(g)$ and $T(g)$.
Write a function for the profit for the month if g gallons are produced and graph this function.
How much milk needs to be produced for the dairy to break even for the month?

The following is a graph of the cost and revenue.



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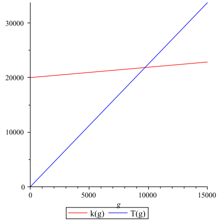
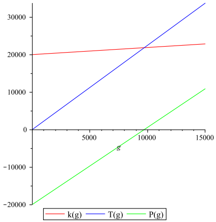


The following graph also includes the profit



Definition

Given two functions f and g we can form the composition function $g \circ f(x) = g(f(x))$ if the output values of f can always be used as input values for g. It must be the case also that the unit measure of the output values of f are identical to the input units of g.

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Suppose that $C(p) = \sqrt{p}$ parts per million and that $p(t) = 0.4t^2 + 2.5$ thousand people t years after 1980 ($t \geq 0$).
Write a function that gives the lake contamination as a function of time.

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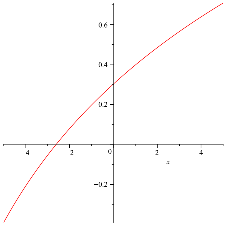
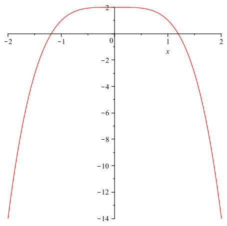
Horizontal Line Test

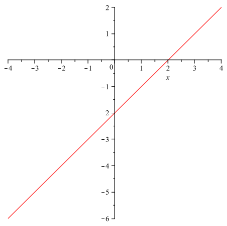
A function is one-to-one if any horizontal line meets the graph of the function in at most one place. If there is a horizontal line meeting the function in more than one place then the function is not one-to-one.

Example

Which of the following are graphs of one-to-one functions?







Definition

For a one-to-one function $g(x)$, the function $f(x)$ is an <u>inverse</u> of g if f sends the output values of g to the corresponding input values. That is,

$$f(g(x)) = x \text{ and } g(f(x)) = x.$$

Example

The underwater pressure measured in atmospheres (atm) d feet below the surface is given numerically by the following table

Depth (ft)	0	33	66	99	132
Pressure (atm)	1	2	3	4	5

This data has the linear model

$$p(d) = \frac{1}{33}d + 1 \text{ atm.}$$

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$$d(p) = 33p - 33 \text{ feet.}$$

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Composition Property of Inverse Functions

Recall that if f and g are inverse functions then

$$f(g(x)) = (f \circ g)(x) = x,$$

and

$$g(f(x)) = (g \circ f)(x) = x.$$