MTHSC 102 Section 1.8 – Logarithmic Functions and Models

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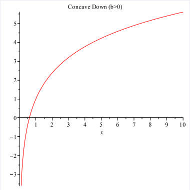

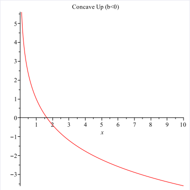

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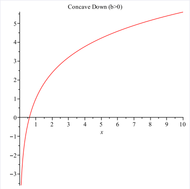

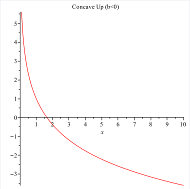

Note

Logarithms do NOT have horizontal asymptotes.

Note

Suppose that $f(x) = a + b\ln(x)$.

b>0 In this case, f has the following properties.

- Increasing
- Concave Down
- $\lim_{x\to 0^+} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

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b<0 In this case, f has the following properties.

- Decreasing
- Concave Up
- $\lim_{x\to 0^+} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$.

Example (Finding a log model)

An international investment fund manager models bond rates of countries. He uses the following data

Time to Maturity	1	2	3	4	5	6	7	8	9	10
Rate	3.60	4.10	4.25	4.40	4.50	4.65	4.75	4.8	4.9	4.95

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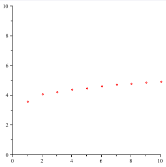

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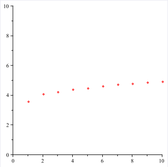

1. Find a log model for the data.
2. What does your model estimate as the rate for 20 year bonds? for 30 year bonds?

Note

Proper alignment of data may be necessary when using log models for 2 reasons.

- Log functions require all input values to be positive.
- Because log functions increase or decrease without bound as inputs approach 0 from the right differently aligned data may give a much better model

So make sure to align the data so that the input values are not near 0. This can be done by simply adding 10 to all of the inputs.

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Note

The inverse of a logarithmic function is an exponential function. Give a logarithmic function, we can find the inverse by plotting a few data points, reversing the input and output data and then using our calculator's exponential regression function.