MTHSC 102 Section 1.10 – Logistic Functions and Models

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Definition

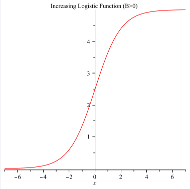
Verbally A logistic function is a function which initially experiences exponential growth or decay, but has bounded output over its lifetime.

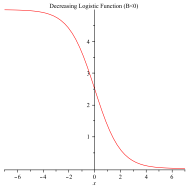
Algebraically A logistic model has an equation of the form

$$f(x) = \frac{L}{1+Ae^{-Bx}},$$

where $A > 0$ and B are nonzero constants and $L > 0$ is the limiting value of the function.

Graphically A logistic function has one of the following forms.





Behavior

Suppose that $f(x) = \frac{L}{1+Ae^{-Bx}}$ is a logistic function. Then

B>0
- f is increasing.
- f begins concave up and then changes to concave down.
- $\lim_{x\to-\infty} f(x) = 0$.
- $\lim_{x\to\infty} f(x) = L$.

B<0
- f is decreasing.
- f begins concave down and then changes to concave up.
- $\lim_{x\to-\infty} f(x) = L$.
- $\lim_{x\to\infty} f(x) = 0$.

In both cases, there is a single inflection point.

Example

The following table shows the number of bacteria present in a biology experiment *d* days after the beginning of the experiment.

Day	1	2	3	4	5	6	7	8	9
Amount	4	15	52	165	391	619	733	771	782

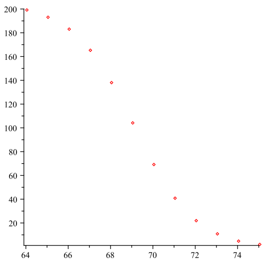
1. Find a logistic model that fits the data.
2. What is the end behavior of the model as time increases.

Example

Of a group of 200 college men surveyed, the number who were taller than a given number of inches is recorded below.

Inches	64	65	66	67	68	69	70	71	72	73	74	75
Number of Men	200	194	184	166	139	105	70	42	23	12	6	3

A scatter plot of the data looks like



Align the input data by subtracting 65 and give a model for the resulting data.