MTHSC 102 Section 1.11 – Cubic Functions and Models

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Definition

Verbally A cubic function is a function whose third differences are constant. It can achieve either a local max and a local min or neither. It has one inflection point.

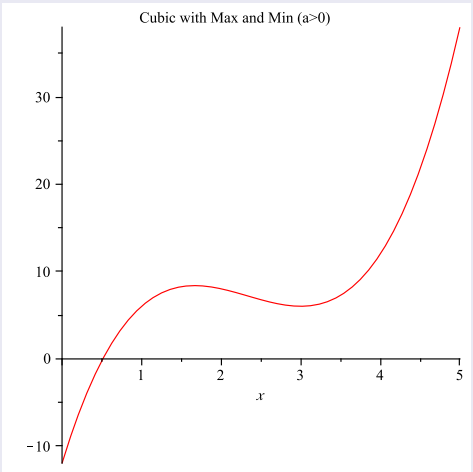
Algebraically A cubic model has an equation of the form

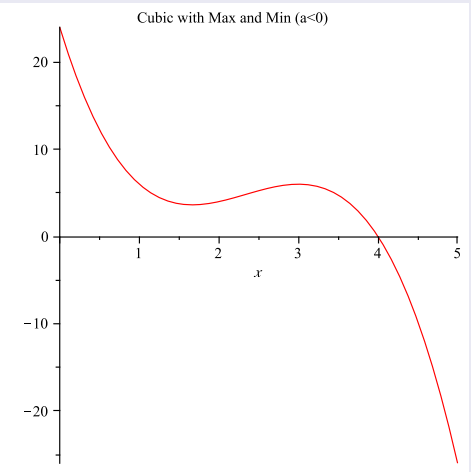
$$f(x) = ax^3 + bx^2 + cx + d,$$

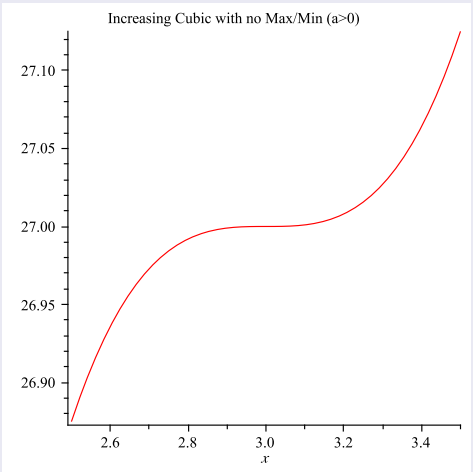
where $a \neq 0$ is a constant and b, c and d are constants.

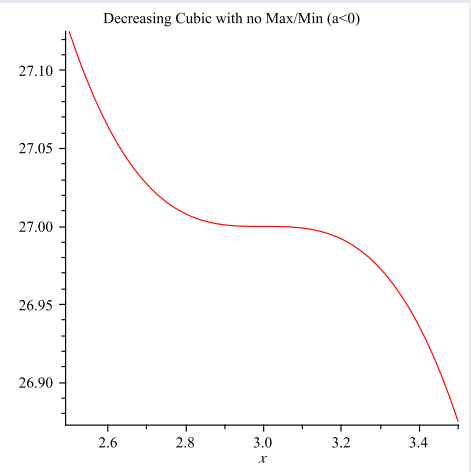
Definition Continued ...

Graphically A cubic function has one of the following forms.









Note

Suppose that $f(x) = ax^3 + bx^2 + cx + d$ is a cubic function. Then,

$a>0$
- f begins and ends increasing.
- f is concave down and then concave up.
- $\lim_{x\to\infty} f(x) = \infty$.
- $\lim_{x\to-\infty} f(x) = -\infty$

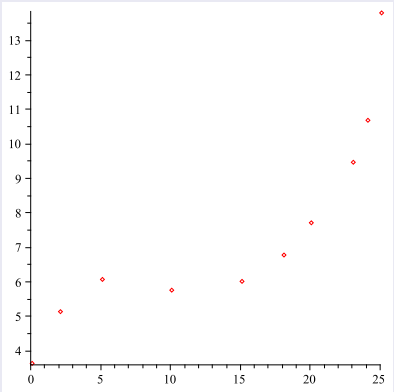
$a<0$
- f begins and ends decreasing.
- f is concave up and then concave down.
- $\lim_{x\to\infty} f(x) = -\infty$.
- $\lim_{x\to-\infty} f(x) = \infty$.

Example

The average price in dollars per 1000 cubic feet of natural gas for residential use in the US for selected years from 1980 through 2005 is given in the following table.

Year	1980	1982	1985	1990	1995	1998	2000	2003	2004	2005
Price	3.68	5.17	6.12	5.80	6.06	6.82	7.76	9.52	10.74	13.84

Plotting this data, we have the following scatter plot.



1. Find a cubic model for this data.
2. Would it be wise to use this model to predict future gas prices?
3. Estimate the price in 1993.

Choosing a Model

1. Examine a scatter plot of your data.
   1. If the plot appears to be near a straight line, then try a linear model.
   2. If the data appears to lie on a curve and there is no inflection point try an exponential, log or quadratic model. It may be helpful to consider 2nd differences and first percentage changes and end behavior.
   3. If there appears to be an inflection point try a cubic or logistic model.
2. Look at the fit of the (at most 2) possible models from the first step.
3. Look at the end behavior. Perhaps you can discern between models not separated above by end behavior.
4. Consider that thee may be two equally good choices of model.