

MTHSC 102 SECTION 2.2 – MEASURES OF CHANGE AT A POINT

Kevin James

DEFINITION

The instantaneous rate of change at a point on a curve is the slope of the graph at that point.

LOCAL LINEARITY

If we look closely enough near any point P on a smooth curve, the curve will be very close to the line tangent to the curve at P . That is, the tangent line is a very good approximation of the curve near P .

DEFINITION

The slope of a smooth graph at a point P is the slope of the line which is tangent to the graph at P .

LINE TANGENT TO A SMOOTH CURVE

The tangent line at a point Q on a smooth continuous graph is the limiting position of the secant lines between point Q and a point P as P approaches Q along the graph (provided the limiting position exists).

GENERAL RULE FOR TANGENT LINES

Lines tangent to a smooth nonlinear curve typically lie on one side or the other of the graph. Tangent lines only cross the graph if the point is an inflection point.

SECANT LINES AND TANGENT LINES

NOTE

Suppose that $f(x)$ is a continuous smooth function. Let T denote the point $(x, f(x))$ for some fixed point x . Let P_n denote the point $(x_n, f(x_n))$ where x_n is a sequence of points approaching x . Then,

$$\lim_{x_n \rightarrow x} \left(\begin{array}{c} \text{slope of} \\ \text{the secant} \\ \text{through } P_n \\ \text{and } T \end{array} \right) = \left(\begin{array}{c} \text{slope of the} \\ \text{graph at } T \end{array} \right) = \left(\begin{array}{c} \text{slope of tan-} \\ \text{gent line at} \\ T \end{array} \right)$$

INTERPRETATION OF THE SLOPE OF A GRAPH

NOTE

Suppose that $f(x)$ is a smooth continuous function. Then,

- If the slope of the graph is positive then the function is increasing.
- If the slope of the graph is negative then the function is decreasing.
- If the slope of the graph is 0 then the graph is locally constant. This could be the location of a local max or min.
- The absolute value of the slope is a measure of the steepness of the graph.

RATE OF CHANGE AND PERCENTAGE RATES OF CHANGE

DEFINITION

Suppose that $f(x)$ is a smooth continuous function. The rate of change of f at x is denoted by $f'(x)$ and is defined to be the slope of the graph of $f(x)$ at $(x, f(x))$ unless the tangent line at $(x, f(x))$ is vertical.

DEFINITION

If $f'(x)$ is defined and $f(x) \neq 0$, then we define

$$\text{percentage rate of change of } f \text{ at } x = \frac{f'(x)}{f(x)} \cdot 100\%$$

The units for this quantity are % per 1 input unit.