MTHSC 102 Section 2.2 – Measures of Change at a Point

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Definition

The instantaneous rate of change at a point on a curve is the slope of the graph at that point.

Local Linearity

If we look closely enough near any point P on a smooth curve, the curve will be very close to the line tangent to the curve at P. That is, the tangent line is a very good approximation of the curve near P.

Definition

The slope of a smooth graph at a point P is the slope of the line which is tangent to the graph at P.

Line Tangent to a Smooth Curve

The tangent line at a point Q on a smooth continuous graph is the limiting position of the secant lines between point Q and a point P as P approaches Q along the graph (provided the limiting position exists).

General Rule for Tangent Lines

Lines tangent to a smooth nonlinear curve typically lie on one side or the other of the graph. Tangent lines only cross the graph if the point is an inflection point.

Note

Suppose that $f(x)$ is a continuous smooth function. Let T denote the point $(x, f(x))$ for some fixed point x. Let P_n denote the point $(x_n, f(x_n))$ where x_n is a sequence of points approaching x. Then,

$$\lim_{x_n \to x} \begin{pmatrix} \text{slope of} \\ \text{the secant} \\ \text{through } P_n \\ \text{and } T \end{pmatrix} = \begin{pmatrix} \text{slope of the} \\ \text{graph at T} \end{pmatrix} = \begin{pmatrix} \text{slope of tan-} \\ \text{gent line at} \\ \text{T} \end{pmatrix}$$

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- If the slope of the graph is negative then the function is decreasing.
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- The absolute value of the slope is a measure of the steepness of the graph.

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Definition

If $f'(x)$ is defined and $f(x) \neq 0$, then we define

$$\underline{\text{percentage rate of change of } f \text{ at } x} = \frac{f'(x)}{f(x)} \cdot 100\%$$

The units for this quantity are % per 1 input unit.