

MTHSC 102 SECTION 2.3 – RATES OF CHANGE NOTATION AND INTERPRETATION

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AVERAGE VS. INSTANTANEOUS RATES OF CHANGE

Average Rates of Change	Instantaneous Rates of Change
Measures the rate of change of a quantity over an interval	Measures the rate of change of a quantity at a point
Slope of a secant line	Slope of the tangent line
Requires data points or a continuous curve to calculate	Requires a continuous smooth curve to calculate

EQUIVALENT TERMINOLOGY

All of the following phrases mean the same.

- Instantaneous rate of change
- rate of change
- slope of the curve
- slope of the tangent line
- **derivative**

DERIVATIVE NOTATION

We have several notations for the derivative of $f(t)$ with respect to t , namely $\frac{df}{dt}$, $f'(t)$, $\frac{d}{dt} [f(t)]$.

Note that here f is the output variable (or function) and t is the input variable.

INTERPRETING DERIVATIVES

When discussing instantaneous rate or change at a point (or the derivative of a function at a point), be sure to include the following information.

- 1 Specify the input value.
- 2 Specify the quantity that is changing.
- 3 Indicate whether the change is a decrease or increase.
- 4 Give the numerical answer labeled with proper units.
- 5 The units for the derivative should be the output units per one input unit (as for average rate of change).

GRAPHS FROM DERIVATIVES

NOTE

We can tell a lot about the graph of a smooth continuous function from the values of its derivative.

- Where $f'(x) = 0$, the graph of $f(x)$ has a horizontal tangent line.
- Where $f'(x) > 0$, the graph of $f(x)$ is increasing and the steepness is given by $f'(x)$.
- Where $f'(x) < 0$, the graph of $f(x)$ is decreasing and the steepness is given by $|f'(x)|$.
- The point of most rapid increase/decrease of $f(x)$ (-i.e. the max/min of $f'(x)$) occurs at an inflection point of the graph of $f(x)$.
 - 1 To the left of the point of most rapid increase the graph of $f(x)$ is concave up. To the right of the point of most rapid increase the graph of $f(x)$ will be concave down.
 - 2 To the left of the point of most rapid decrease, the graph of $f(x)$ is concave down. To the right of the point of most rapid decrease, the graph of $f(x)$ is concave up.