# Average vs. Instantaneous Rates of Change

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All of the following phrases mean the same.

- Instantaneous rate of change
- Rate of change
- Slope of the curve
- Slope of the tangent line
- Derivative
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Derivative Notation

We have several notations for the derivative of $f(t)$ with respect to $t$, namely $\frac{df}{dt}$, $f'(t)$, $\frac{d}{dt} [f(t)]$.

Note that here $f$ is the output variable (or function) and $t$ is the input variable.

Interpreting Derivatives

When discussing instantaneous rate or change at a point (or the derivative of a function at a point), be sure to include the following information.

1. Specify the input value.
2. Specify the quantity that is changing.
3. Indicate whether the change is a decrease or increase.
4. Give the numerical answer labeled with proper units.
5. The units for the derivative should be the output units per one input unit (as for average rate of change).
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**Note**

We can tell a lot about the graph of a smooth continuous function from the values of its derivative.

- Where $f'(x) = 0$, the graph of $f(x)$ has a horizontal tangent line.
- Where $f'(x) > 0$, the graph of $f(x)$ is increasing and the steepness is given by $f'(x)$.
- Where $f'(x) < 0$, the graph of $f(x)$ is decreasing and the steepness is given by $|f'(x)|$.
- The point of most rapid increase/decrease of $f(x)$ (i.e., the max/min of $f'(x)$) occurs at an inflection point of the graph of $f(x)$.

1. To the left of the point of most rapid increase, the graph of $f(x)$ is concave up. To the right of the point of most rapid increase, the graph of $f(x)$ will be concave down.

2. To the left of the point of most rapid decrease, the graph of $f(x)$ is concave down. To the right of the point of most rapid decrease, the graph of $f(x)$ is concave up.
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