MTHSC 102 Section 2.3 – Rates of Change Notation and Interpretation

Kevin James

Average vs. Instantaneous Rates of Change

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Equivalent Terminology

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- **derivative**

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5. The units for the derivative should be the output units per one input unit (as for average rate of change).

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  2. To the left of the point of most rapid decrease, the graph of $f(x)$ is concave down. To the right of the point of most rapid decrease, the graph of $f(x)$ is concave up.