MTHSC 102 Section 2.4 – Rates of Cahnge–Numerical Limits and Nonexistence

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Note

The line secant to $y = f(x)$ and passing through $P = (a, f(a))$ and $Q = (b, f(b))$ has slope

$$\text{slope} = \frac{f(b) - f(a)}{b - a}$$

Estimating the slope of the tangent line

1. Calculate the slope of the secant line passing through P and several nearby points $Q_i = (b_i, f(b_i))$ $(i = 1, 2, 3, \dots)$ to the left of P.
2. Does the slope seem to be getting close to some value as Q_i approaches P? If so, what value?
3. Calculate the slope of the secant line passing through P and several nearby points $Q_i = (b_i, f(b_i))$ $(i = 1, 2, 3, \dots)$ to the right of P.
4. Does the slope seem to be getting close to some value as Q_i approaches P? If so, what value?
5. If you obtained values for 2 and 4 and if they are the same, their common value is a good estimate for the slope of the tangent line.

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Example

A company invests \$32 billiion of its assets electronically in the global market, resulting in an investment with continuous compounding at 12% per year APY.
Note that the value of such an investment after t years is given by $A(t) = 32 \cdot (1.12)^t$ billion dollars.

1. How rapidly is the investment growing at the middle of the fifth year (-i.e. $t = 4.5$)?
2. At what percentage rate of change is this investment growing?

Definition

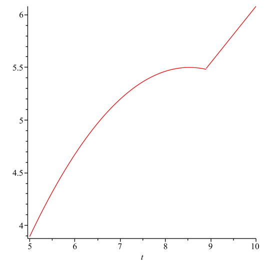
A piecewise continuous function is a function that is continuous over different intervals but has a break point. It is often defined by different equations over different intervals.

Example

For example the function

$$f(x) = \begin{cases} -0.129t^2 + 2.25 - 3.88 & \text{when } 5 \leq t \leq 9, \\ 0.536t + 0.72 & \text{when } 9 \leq t \leq 10. \end{cases}$$

has the following graph.



At the point $(9, f(9))$, the tangent line is not well defined.

Definition

We call a point P on the graph of a continuous function f a sharp point when the secant lines joining P to close points on either side of it have different limiting positions.

Note

If a function is not continuous or if its graph has a sharp point at $(a, f(a))$, the rate of change (or derivative) does not exist at the point $(a, f(a))$.
If a continuous function has a point $(a, f(a))$ where the tangent line is vertical, the rate of change does not exist at $(a, f(a))$.

Definition

A function f is differentiable at P if the instantaneous rate of change (derivative) of f exists at P.
A function f is differentiable on an open interval if it is differentiable at each point in the interval.