MTHSC 102 Section 2.4 – Rates of Cahnge–Numerical Limits and Nonexistence

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Note

The line secant to $y = f(x)$ and passing through $P = (a, f(a))$ and $Q = (b, f(b))$ has slope

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5. If you obtained values for 2 and 4 and if they are the same, their common value is a good estimate for the slope of the tangent line.

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1. How rapidly is the investment growing at the middle of the fifth year (-i.e. $t = 4.5$)?
2. At what percentage rate of change is this investment growing?

Definition

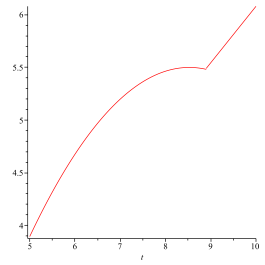
A piecewise continuous function is a function that is continuous over different intervals but has a break point. It is often defined by different equations over different intervals.

Example

For example the function

$$f(x) = \begin{cases} -0.129t^2 + 2.25 - 3.88 & \text{when } 5 \leq t \leq 9, \\ 0.536t + 0.72 & \text{when } 9 \leq t \leq 10. \end{cases}$$

has the following graph.



At the point $(9, f(9))$, the tangent line is not well defined.

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If a function is not continuous or if its graph has a sharp point at $(a, f(a))$, the rate of change (or derivative) does not exist at the point $(a, f(a))$.
If a continuous function has a point $(a, f(a))$ where the tangent line is vertical, the rate of change does not exist at $(a, f(a))$.

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A function f is differentiable on an open interval if it is differentiable at each point in the interval.