MTHSC 102 Section 2.5 – Rates of Change Defined over Intervals

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Four Step Method to Find $f'(x)$

Given a function f, the equation for the derivative with respect to x can be found as follows.

1. Begin with a typical point $(x, f(x))$.
2. Choose a close point $(x+h, f(x+h))$.
3. Write a formula for the slope of the secant line between the two points
$$\text{slope} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}.$$
4. Make sure to simplify the formula for slope as much as possible.
5. Evaluate
$$\lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right].$$

This limiting value is the derivative formula at each input where the limit exists.

Recall

1. $\lim_{x \to a^-} f(x)$ denotes the value that $f(x)$ approaches as x approaches (but is not equal to) a from the left.
2. $\lim_{x \to a^+} f(x)$ denotes the value that $f(x)$ approaches as x approaches (but is not equal to) a from the right.
3. $\lim_{x \to a} f(x)$ denotes the common value of the previous two **ONLY** when they both exist and they are the same.

Note

If the numerator and denominator of a rational function share a common factor, then the function obtained by algebraically canceling the common factor has all limits identical to those of the original function.

Example

Suppose that $f(x) = x^2 - 15x + 6$.

1. Find $\left.\frac{df}{dx}\right|_{x=2}$.
2. Find a formula for the derivative $f'(x)$ at an arbitrary point x.
3. Give a description of the derivative.

Derivative Formula

If $y = f(x)$, then the derivative $\frac{\mathrm{dy}}{\mathrm{dx}}, \frac{\mathrm{df}}{\mathrm{dx}}, f'(x)$ is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example

Compute the derivative of the function $f(x) = 2\sqrt{x}$.