MTHSC 102 Section 2.5 – Rates of Change Defined over Intervals

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Four Step Method to Find $f'(x)$

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3. Write a formula for the slope of the secant line between the two points

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This limiting value is the derivative formula at each input where the limit exists.

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Note

If the numerator and denominator of a rational function share a common factor, then the function obtained by algebraically canceling the common factor has all limits identical to those of the original function.

Example

Suppose that $f(x) = x^2 - 15x + 6$.

1. Find $\left.\frac{\mathrm{df}}{\mathrm{dx}}\right|_{x=2}$.
2. Find a formula for the derivative $f'(x)$ at an arbitrary point x.
3. Give a description of the derivative.

Derivative Formula

If $y = f(x)$, then the derivative $\frac{\mathrm{dy}}{\mathrm{dx}}$, $\frac{\mathrm{df}}{\mathrm{dx}}$, $f'(x)$ is given by

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Example

Compute the derivative of the function $f(x) = 2\sqrt{x}$.