MTHSC 102 Section 2.6 – Rate of Change Graphs

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Definition

Suppose that $y = f(x)$ is a smooth continuous curve. The graph of $y = f'(x)$ is called the slope graph for this curve.

The graphs of a function and its derivative.

Suppose that $f(x)$ is a function and that f' is its derivative. The graphs of f and f' are related as follows.

1. The points on the graph of f where the tangent line is horizontal have inputs x for which $f'(x)$ is 0.
2. The input intervals in which f is increasing are the input intervals where f' will be positive.
3. The input intervals in which f is decreasing are the input intervals where f' will be negative.
4. The inflection points on the graph of the function have input values for which the graph of f' flattens and may change direction and may be points where f' is undefined due to a vertical tangent line.
5. Note also that where f is c.c.u f' will be increasing and where f is c.c.d. f' will be decreasing.
6. If the graph of f approaches an horizontal asymptote, f' will approach 0 on the same input intervals.

Note

Given a graph we can draw a few tangent lines and estimate the slopes in order to plot some points on the graph of the derivative graph. That is, if the slope of the line tangent to $y = f(x)$ at a is m then $f'(a) = m$. Thus, the point (a, m) is on the graph of $f'(x)$.

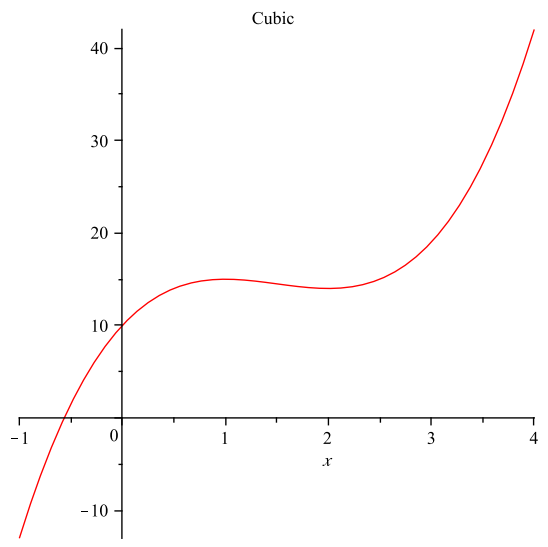
Points at which the derivative does not exist

Suppose that $f(x)$ is a function. Then f' will not be defined at a point a under the following conditions.

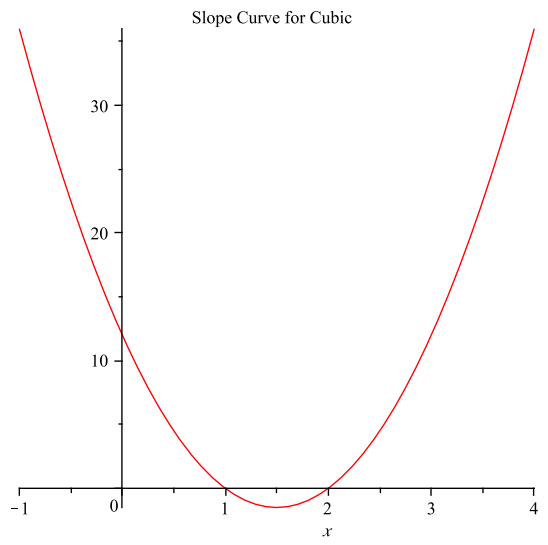
1. If f is not defined at a, then $f'(a)$ is not defined.
2. If f is defined at a but not continuous there, then $f'(a)$ is not defined.
3. If f is defined and continuous at a but has a sharp point (-i.e. the left and right hand limits of the slopes of secant lines do not agree) at a then f' is not defined at a. In fact, if f is defined and continuous at a and both one sided limits exist but disagree at a, then f' has a jump discontinuity at a.
4. If the tangent line to $y = f(x)$ at a is vertical, then f' does not exist.

Example

The graph

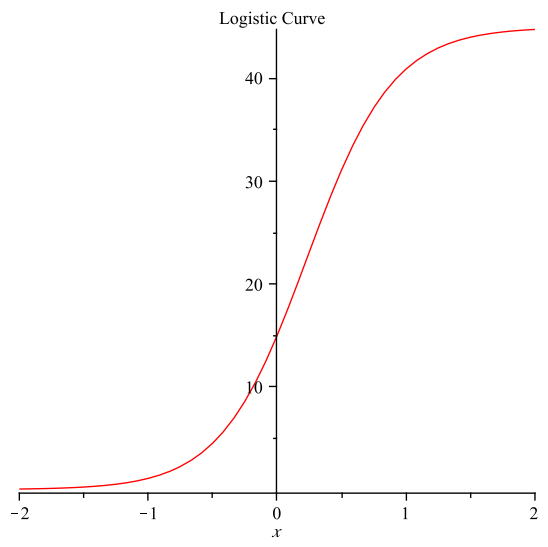


has the slope graph



Example

Consider the logistic curve.



It has the slope graph