MTHSC 102 Section 2.6 – Rate of Change Graphs

Kevin James

Definition

Suppose that $y = f(x)$ is a smooth continuous curve. The graph of $y = f'(x)$ is called the slope graph for this curve.

The graphs of a function and its derivative.

Suppose that $f(x)$ is a function and that f' is its derivative. The graphs of f and f' are related as follows.

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4. The inflection points on the graph of the function have input values for which the graph of f' flattens and may change direction and may be points where f' is undefined due to a vertical tangent line.

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5. Note also that where f is c.c.u f' will be increasing and where f is c.c.d. f' will be decreasing.

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5. Note also that where f is c.c.u f' will be increasing and where f is c.c.d. f' will be decreasing.
6. If the graph of f approaches an horizontal asymptote, f' will approach 0 on the same input intervals.

Note

Given a graph we can draw a few tangent lines and estimate the slopes in order to plot some points on the graph of the derivative graph. That is, if the slope of the line tangent to $y = f(x)$ at a is m then $f'(a) = m$. Thus, the point (a, m) is on the graph of $f'(x)$.

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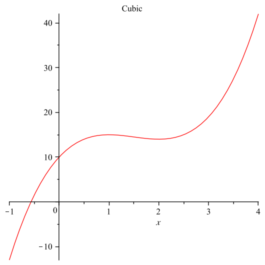
4. If the tangent line to $y = f(x)$ at a is vertical, then f' does not exist.

EXAMPLE

The graph



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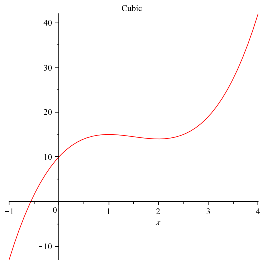

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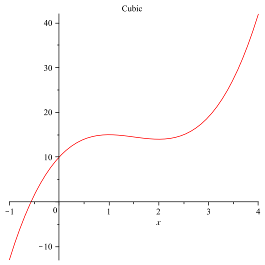

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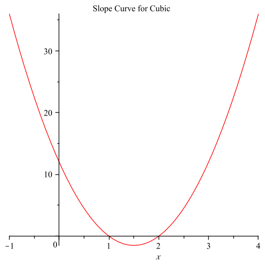

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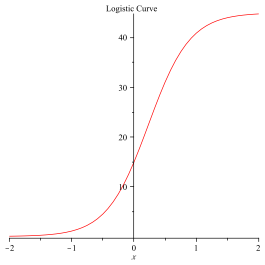

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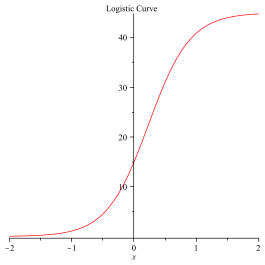

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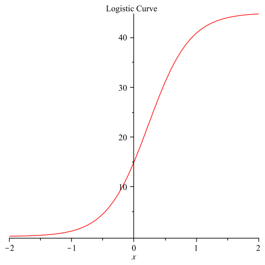

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