MTHSC 102 Section 3.3-4 – The Chain Rule

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$$\frac{dC}{dt} = \left(\frac{dC}{dp}\right)\left(\frac{dp}{dt}\right)$$

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2. Calculate the rate of change with respect to time of the average cost for violins in 2008.

The Chain Rule (2nd form)

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$$\frac{\mathrm{df}}{\mathrm{dx}} = f'(x) = h'(g(x)) \cdot g'(x).$$

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Write the derivatives with respect to x of the following functions.

1. $y = e^{x^2}$
2. $y = (x^3 + 2x^2 + 4)^{\frac{1}{2}}$
3. $y = \frac{3}{4-2x^2}$
4. $y = \frac{24}{1+0.04e^{0.6x+0.2}} + 52$