MTHSC 102 Section 3.5-6 – The Product Rule

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2. Suppose that during the 2007-2008 school year, tuition was \$ 9,000 and that enrollment was 18,503 students. Suppose also that tuition was increasing at a rate of \$ 500 per year and that the enrollment at the University was decreasing at a rate of 50 students per year. What was the total revenue in during this year?

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3. What was the rate of change in total revenue at this time?

Theorem (Product Rule)

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$$\frac{\mathrm{dR}}{\mathrm{dt}} = \frac{\mathrm{dT}}{\mathrm{dt}} \cdot E(t) + T(t) \cdot \frac{\mathrm{dE}}{\mathrm{dt}}.$$

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1. What equation models the revenue generated by CD sales during a 4 week period when the price is x dollars per CD?
2. What is the rate of change in revenue when the price is \$ 10, \$ 12, \$ 15.

Theorem (Quotient Rule)

Suppose that $f(x) = \frac{g(x)}{h(x)}$. *Then the derivatives are related by*

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Note

The quotient rule can be achieved by applying the product and chain rules to $f(x) = g(x) \cdot [h(x)]^{-1}$.