MTHSC 102 Section 4.1 – Approximating Change

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NOTE

The approximate change in f is caused by changing x to $x + h$ for h, small can be approximated by

$$f(x + h) - f(x) \approx f'(x) \cdot h.$$

FACT

When the input x changes by a small amount h to $x + h$, the output $f(x + h)$ can be approximated by

$$f(x + h) \approx f(x) + f'(x) \cdot h.$$

EXAMPLE

The temperature for a 2 hour period during and after a thunderstorm can be modeled by

$$T(t) = 2.37t^4 - 5.163t^3 + 8.69t^2 - 9.87t + 78 \text{ }^\circ\text{F},$$

where t is the number of hours since the storm began.

① Use the rate of change of $T(t)$ to at $t = 0.25$ to estimate by how much the temperature changed between 15 and 20 minutes after the storm began.

② Find the temperature and rate of change of temperature at $t = 1.5$ hours.

③ Using only the answers to the last question, estimate the temperature 1 hour and 40 minutes after the storm began.

DEFINITION

Suppose that f is a smooth continuous function. We define the linearization of f at the value c as

$$f_L(x) = f(c) + f'(c)(x - c).$$

NOTE

Suppose that f is a smooth continuous function and that f_L is its linearization at c.

1. $f(c) = f_L(c)$.
2. $f(x) \approx f_L(x)$ when x is near c.
3. The above approximation gets worse as x moves away form c.

The number of full-time employees over the previous six year period at a certain company whose ages are between 20 and 24 can be modeled by

$$f(t) = -12.92t^3 + 185.45t^2 - 729.35t + 10038.57 \quad \text{thousand employees}$$

where t is the number of years from the beginning of the six year period. Note that $t = 7$ corresponds to the present and that the data was observed over the period $1 \leq t \leq 7$.

1. Find a linear model for the future number of employees from the values of $f'(7)$ and $f(7)$.

2. Use the linear model to predict the number of full-time employees whose ages are between 20 and 24 during the next two years.