MTHSC 102 Section 4.1 – Approximating Change

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Note

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Fact

When the input x changes by a small amount h to $x+h$, the output $f(x+h)$ can be approximated by

$$f(x+h) \approx f(x) + f'(x) \cdot h.$$

Example

The temperature for a 2 hour period during and after a thunderstorm can be modeled by

$$T(t) = 2.37t^4 - 5.163t^3 + 8.69t^2 - 9.87t + 78\ {}^\circ\text{F},$$

where t is the number of hours since the storm began.

1. Use the rate of change of $T(t)$ to at $t = 0.25$ to estimate by how much the temperature changed between 15 and 20 minutes after the storm began.

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2. Find the temperature and rate of change of temperature at $t = 1.5$ hours.
3. Using only the answers to the last question, estimate the temperature 1 hour and 40 minutes after the storm began.

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1. $f(c) = f_L(c)$.
2. $f(x) \approx f_L(x)$ when x is near c.
3. The above approximation gets worse as x moves away form c.

Example

The number of full-time employees over the previous six year period at a certain company whose ages are between 20 and 24 can be modeled by

$$f(t) = -12.92t^3 + 185.45t^2 - 729.35t + 10038.57 \quad \text{thousand employees}$$

where t is the number of years from the beginning of the six year period. Note that $t = 7$ corresponds to the present and that the data was observed over the period $1 \leq t \leq 7$.

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1. Find a linear model for the future number of employees from the values of $f'(7)$ and $f(7)$.
2. Use the linear model to predict the number of full-time employees whose ages are between 20 and 24 during the next two years.