MTHSC 102 Section 4.2 – Relative Extreme Points

Kevin James

Definition (Relative Extreme Points and Relative Extreme Values)

Suppose that $f(x)$ is a function defined on an interval I.

1. We say that f attains a <u>relative maximum value</u> of $f(a)$ at $x = a$ if there is some interval (b, c) with $b < a < c$ and such that for all $x \in (b, c)$, $f(x) \leq f(a)$.
   In this case, the point $(a, f(a))$ is called a <u>relative maximum point</u>.
2. We say that f attains a <u>relative minimum value</u> of $f(a)$ at $x = a$ if there is some interval (b, c) with $b < a < c$ and such that for all $x \in (b, c)$, $f(x) \geq f(a)$.
   In this case, the point $(a, f(a))$ is called a <u>relative minimum point</u>.

Fact

If f is a smooth continuous function and if f attains a relative extreme value at $x = a$, then the derivative f' crosses the input axis at $x = a$ and thus $f'(a) = 0$.

Definition

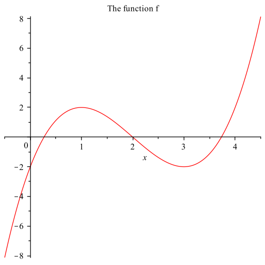
A critical point of a continuous function f is a point $(c, f(c))$ for which either $f'(c)$ does not exist or for which $f'(c) = 0$.
The input value c of a critical point $(c, f(c))$ is referred to as a critical input or critical number.

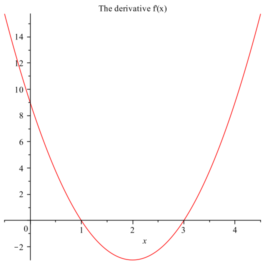
Example

Consider the function

$$f(x) = x^3 - 6x^2 + 9x - 2.$$

The graph of this function and its derivative are





Find all relative extreme points of f.

Note

Relative extrema do not occur if the derivative touches the input axis but does not cross it. That is, $f'(a)$ may be zero even when f does not attain a relative extreme at $x = a$.

First Deivative Test for Relative Extrema

Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then $f(c)$ is a relative maximum value of f.
2. If f' changes from negative to positive at c, then $f(c)$ is a relative minimum value of f.
3. If f' does not change sign at c, then f does not attain a relative extreme value at c.

Conditions For Existence of Relative Extrema

For a function f with input x, a relative extreme can occur at $x = c$ only if $f(c)$ exists. Furthermore,

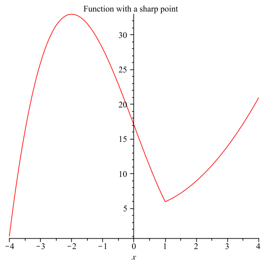
1. A relative extreme exists where $f'(c) = 0$ and the graph of $f'(x)$ crosses the input axis at $x = c$.
2. A relative extreme can exist where $f(c)$ exists but $f'(c)$ does not.

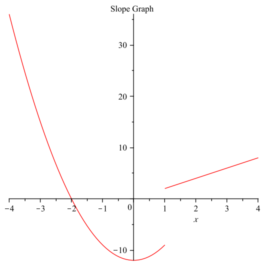
Example

Consider the function

$$f(x) = \begin{cases} x^3 - 12x + 17 & \text{if } x \leq 1, \\ x^2 + 5 & \text{if } x > 1. \end{cases}$$

The graph of this function and its derivative are





Find all relative extreme points of f.

Finding Extrema

To find the relative maxima and minima of a function f,

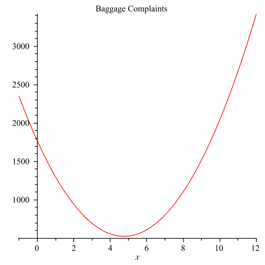
1. Determine the input values for which $f' = 0$ or f' is undefined.
2. Examine a graph of f to determine which of these input values correspond to relative maxima or relative minima.

Example

The number of consumer complaints to the US Department of Transportation about baggage on US airlines between 1989 and 2000 can be modeled by the function

$$B(x) = 55.15x^2 - 524.09x + 1768.65 \text{ complaints},$$

where x is the number of years after 1989.

1. The graph of the function is 

2. Find the relative extrema of B.