MTHSC 102 Section 4.3 – Absolute Extreme Points

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Definition (Relative Extreme Points and Relative Extreme Values)

Suppose that $f(x)$ is a function defined on an interval I (possibly $I = (-\infty, \infty)$.

1. We say that f attains an absolute maximum value on I of $f(a)$ at $x = a$ if for all $x \in I$, $f(x) \leq f(a)$.
   In this case, the point $(a, f(a))$ is called an absolute maximum point.
2. We say that f attains an absolute minimum value on I of $f(a)$ at $x = a$ if for all $x \in I$, $f(x) \geq f(a)$.
   In this case, the point $(a, f(a))$ is called an absolute maximum point.

Absolute Extrema on Closed Intervals

Note

To find the absolute extrema of a continuous function f on a closed interval $[a, b]$:

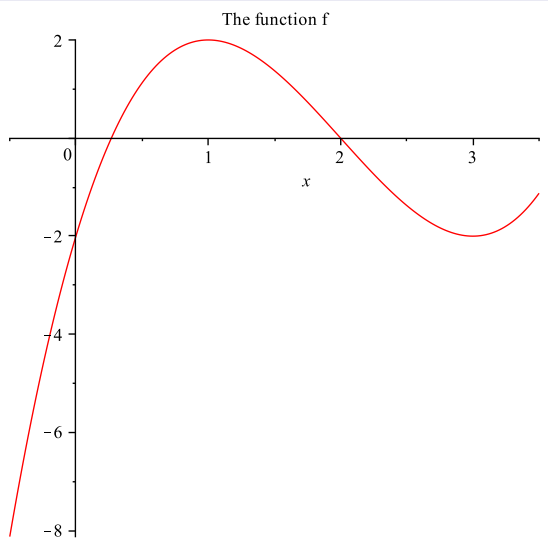
1. Find all relative extrema of f in $[a, b]$.
2. Compute $f(a)$, $f(b)$ and $f(c)$ for all locations $a < c < b$ of relative extrema in $[a, b]$. The largest value is the absolute maximum of f on $[a, b]$ and the smallest value is the absolute minimum of f on $[a, b]$.

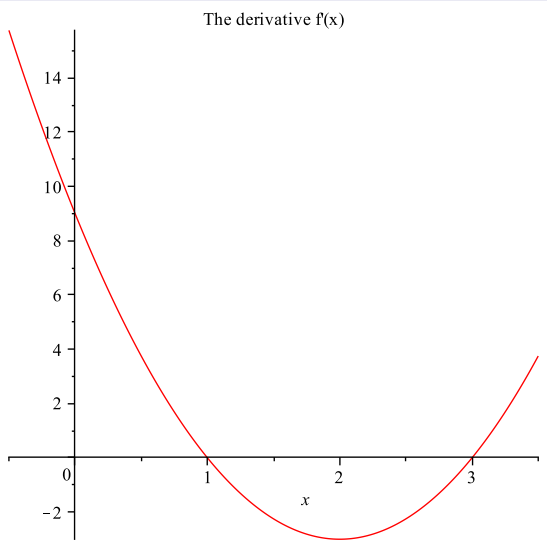
Example

Consider the function

$$f(x) = x^3 - 6x^2 + 9x - 2.$$

The graph of this function and its derivative are





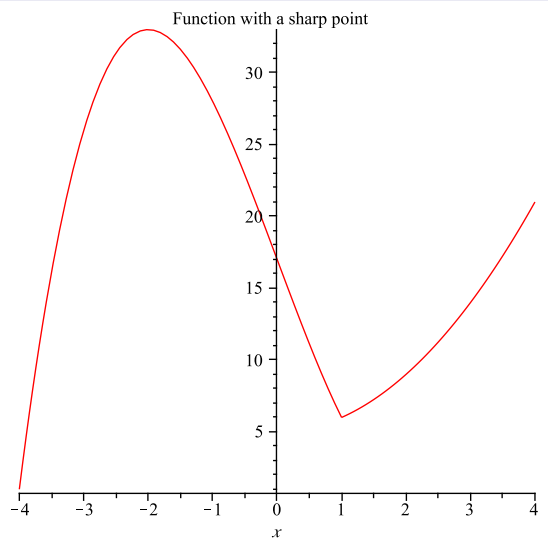
Find all relative and absolute extreme points of f in the interval $[-0.5, 3.5]$.

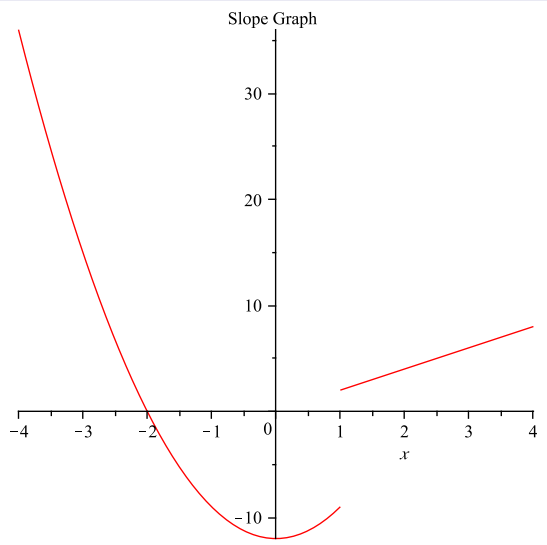
Example

Consider the function

$$f(x) = \begin{cases} x^3 - 12x + 17 & \text{if } x \leq 1, \\ x^2 + 5 & \text{if } x > 1. \end{cases}$$

The graph of this function and its derivative are





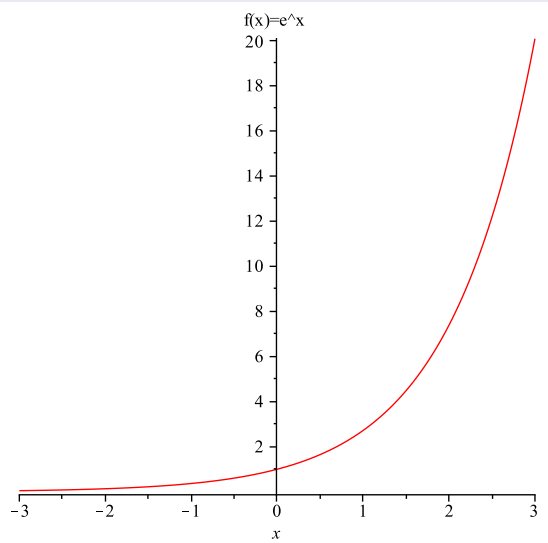
Find all relative and absolute extreme points of f in the interval $[-4, 4]$.

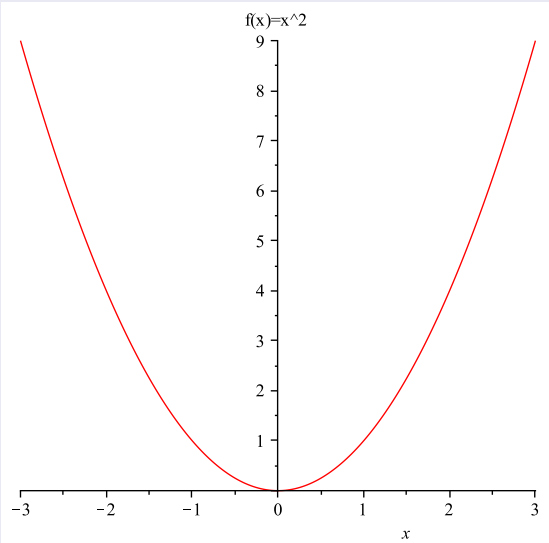
Absolute Extrema for Unbounded Input Values

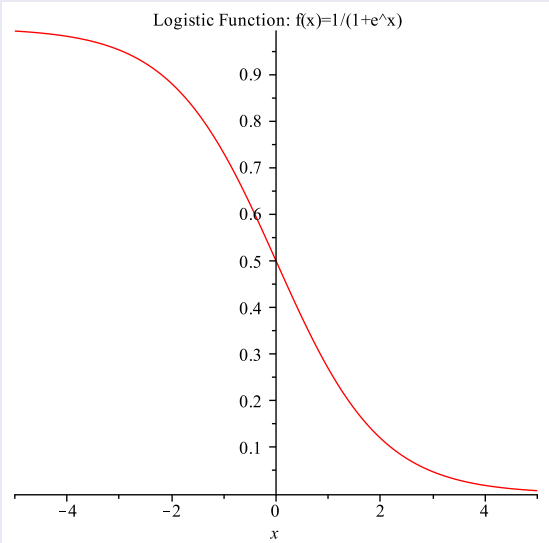
Note

If f is a continuous function and we consider its behavior over all real inputs, it is possible that f does not have a absolute max or absolute min (or both).

Example







Finding Extrema

To find the relative maxima and minima of a function f,

1. Determine the input values for which $f' = 0$ of f' is undefined.
2. Examine a graph of f to determine which of these input values correspond to relative extrema.

To find the absolute maximum and minimum of a function f on an interval $[a, b]$.

1. Find all relative extrema of f in the interval (as above).
2. Compare the relative extreme values in the interval and $f(a)$ and $f(b)$. The largest value is the absolute maximum value and the smallest value is the absoluet minimum.

To find the absolute maximum and minimum of a continuous function f without a specified interval.

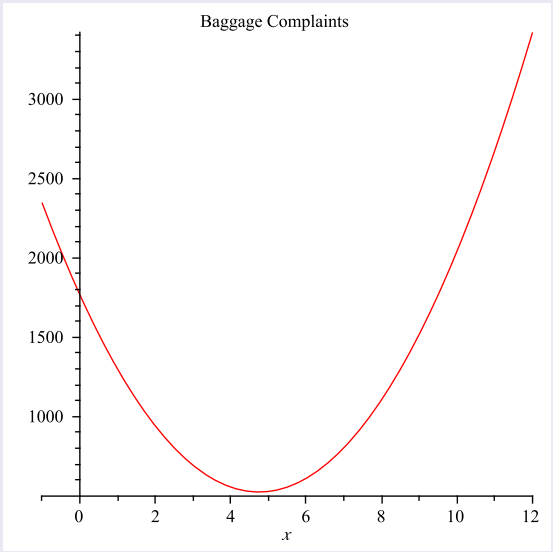
1. Find all relative extrema of f.
2. Determine the end behavior of the function in both directions. The absolute extrema either do not exist or are among the relative extrema.

Example

The number of consumer complaints to the US Department of Transportation about baggage on US airlines between 1989 and 2000 can be modeled by the function

$$B(x) = 55.15x^2 - 524.09x + 1768.65 \text{ complaints},$$

where x is the number of years after 1989.



1. The graph of the function is
2. Find the relative and absolute maxima and minima on the interval $0 \leq x \leq 11$.