MTHSC 102 Section 4.3 – Absolute Extreme Points

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Definition (Relative Extreme Points and Relative Extreme Values)

Suppose that $f(x)$ is a function defined on an interval I (possibly $I = (-\infty, \infty)$).

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1. Find all relative extrema of f in $[a, b]$.
2. Compute $f(a)$, $f(b)$ and $f(c)$ for all locations $a < c < b$ of relative extrema in $[a, b]$. The largest value is the absolute maximum of f on $[a, b]$ and the smallest value is the absolute minimum of f on $[a, b]$.

Example

Consider the function

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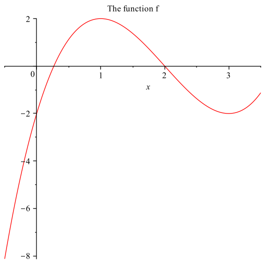

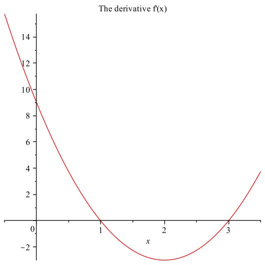

Find all relative and absolute extreme points of f in the interval $[-0.5, 3.5]$.

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$$f(x) = \begin{cases} x^3 - 12x + 17 & \text{if } x \le 1, \\ x^2 + 5 & \text{if } x > 1. \end{cases}$$

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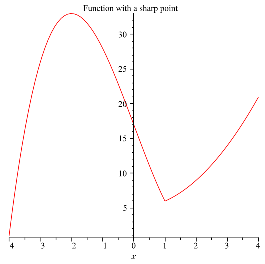

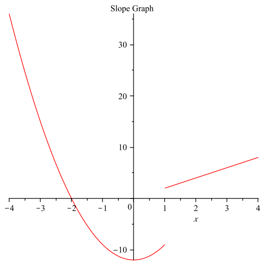

Find all relative and absolute extreme points of f in the interval $[-4, 4]$.

Absolute Extrema for Unbounded Input Values

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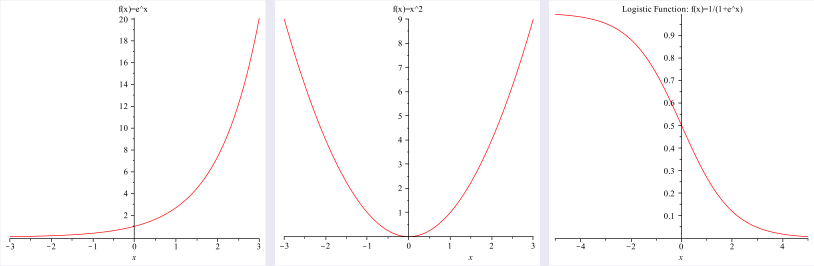

Finding Extrema

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To find the absolute maximum and minimum of a function f on an interval $[a, b]$.

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To find the absolute maximum and minimum of a continuous function f without a specified interval.

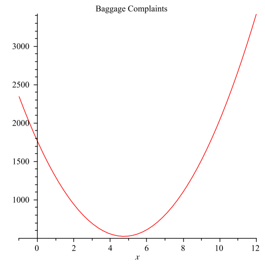
1. Find all relative extrema of f.
2. Determine the end behavior of the function in both directions. The absolute extrema either do not exist or are among the relative extrema.

Example

The number of consumer complaints to the US Department of Transportation about baggage on US airlines between 1989 and 2000 can be modeled by the function

$$B(x) = 55.15x^2 - 524.09x + 1768.65 \text{ complaints},$$

where x is the number of years after 1989.

1. The graph of the function is 

2. Find the relative and absolute maxima and minima on the interval $0 \leq x \leq 11$.