MTHSC 102 Section 4.4 – Inflection Points and Second Derivatives

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Example

A model for the population of KY from 1980-1993 is

$$p(x) = 0.395x^3 - 6.67x^2 + 30.3x + 3661 \quad \text{thousand people}$$

where x is the number of years after 1980.

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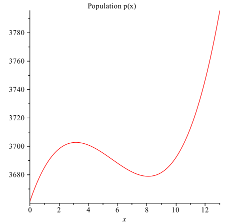
where x is the number of years after 1980.

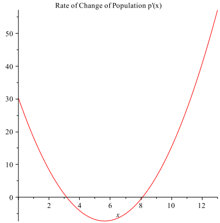
The derivatives of $p(x)$ are

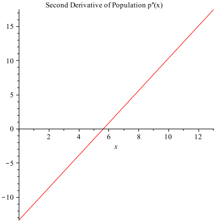
$$p'(x) = 1.185x^2 - 13.34x^+30.3 \quad \text{thousand people per year}$$

$$p''(x) = 2.37x - 13.34 \quad \text{thousand people per year per year}$$

Let's consider the graphs of these functions...


Population p(x)
3780
3760
3740
3720
3700
3680
0
2
4
6
8
10
12
x


Rate of Change of Population p'(x)
50
40
30
20
10
0
2
4
6
8
10
12
x


Second Derivative of Population p''(x)
15
10
5
0
−5
−10
2
4
6
8
10
12
x

RECALL

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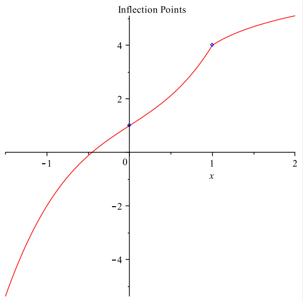
Note

Suppose that f is a smooth continuous function.

1. If $(c, f(c))$ is an inflection point on the graph of f, then $(c, f'(c))$ is a relative extreme point on the graph of f'.
2. If the point $(c, f'(c))$ on the graph of f' is a relative extreme point then $f''(c)$ is either zero or undefined and f'' changes sign at c (if f'' is defined near c).

Example

Consider the graph



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Where are the inflection points? What can you say about the derivatives of this function at the inflection points?

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- That is, they occur at the locations of the relative extreme values of the derivative $f'(x)$.
- The relative extremes of the derivative will occur at the places where the 2nd derivative $f''(x)$ is either 0 or undefined.
- If the 2nd derivative $f''(x)$ is negative on one side of an input value and positive on the other side of that input value, then an inflection point of the function graph occurs at that input value.

Example

The percentage of students graduating from high school in SC from 1982 through 1990 who entered postsecondary institutions can be modeled by

$$f(x) = -0.1057x^3 + 1.355x^2 - 3.672x + 50.792 \quad \%,$$

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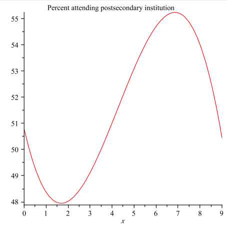

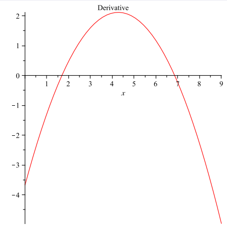

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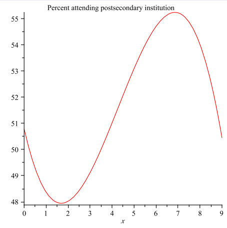
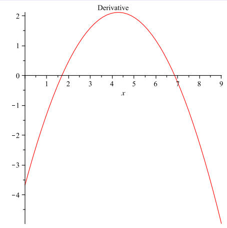
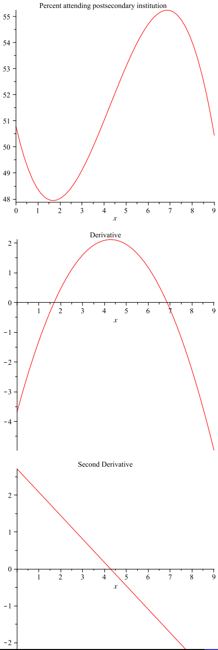




1. Find the inflection point of the function.
2. Determine the year between 1982 and 1990 in which the percentage was increasing most rapidly.
3. Determine the year between 1982 and 1990 in which the percentage was decreasing most rapidly.

Consider again the function from the previous example and its derivatives.



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Second Derivative Test for Relative Extrema

Suppose the function f is continuous over an interval containing c.

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2. If $f'(c) = 0$ and $f''(c) < 0$, then $(c, f(c))$ is a relative maximum point.