

# MTHSC 102 SECTION 4.5 – MARGINAL ANALYSIS

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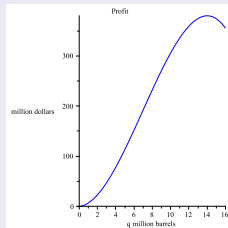
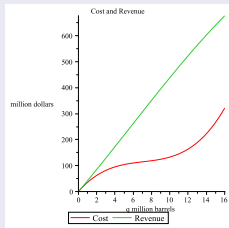
## EXAMPLE

The cost and revenue for the production of  $q$  million barrels of crude oil at a certain company's oil field can be modeled by

$$C(q) = 0.24q^3 - 5.1q^2 + 40.27q + 0.15 \quad \text{million dollars}$$

$$R(q) = -0.03q^3 + 0.52q^2 + 41.69q \quad \text{million dollars,}$$

where  $0 < q < 17$ . We have the following graphs of the cost, revenue and profit.



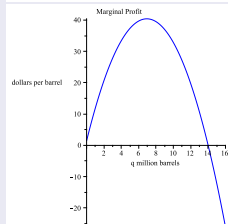
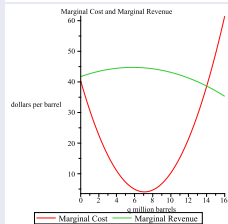
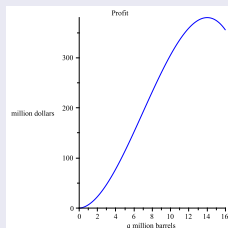
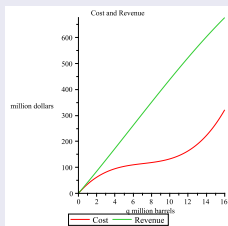
Find the number of barrels to produce in order to maximize profit.

## EXAMPLE CONTINUED ...

In order to maximize profit, we consider the derivative of

$$P(q) = R(q) - C(q).$$

Note that  $P'(q) = R'(q) - C'(q)$ .



## DEFINITION

In Economics, marginal cost is defined as the cost incurred to produce one additional unit.

Marginal revenue is the amount of revenue realized by the sale of one additional unit.

Marginal Profit is the amount of profit realized by the sale of one additional unit.

## NOTE

Suppose that  $C(q)$  and  $R(q)$  are continuous functions which model cost and revenue from the production/sale of  $q$  units.

- 1 Recall that the profit is given by  $P(q) = R(q) - C(q)$ .
- 2  $C'(q)$  gives the marginal cost of producing the  $q + 1$ -st unit.
- 3  $R'(q)$  gives the marginal revenue realized from the sale of the  $q + 1$ -st unit.
- 4  $P'(q) = R'(q) - C'(q)$  gives the marginal profit realized from the sale of the  $q + 1$ -st unit.

## PROFIT MAXIMIZATION RULE

Profit is maximized when marginal cost is equal to marginal revenue.

Note that a proper analysis must be made to ensure that we are not predicting a minimum profit.

## DEFINITION

If  $Q(x)$  is the quantity of a certain product that can be produced when the input is  $x$  ( $x$  could be dollars invested or the size of the labor force, etc.), then we refer to  $Q'(x)$  as the Marginal product. The marginal product measures the change in the production level due to an increase of one unit in the input variable.