MTHSC 102 Section 1.1 – Models and Functions

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Note

We will typically represent related data in four ways or from four viewpoints.

- Numerically (using a chart or table of data)
- Graphically (using a scatter plot or continuous graph)
- Verbally (using a word description)
- Algebraically (using a mathematical model).

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Definition

The process of translating a real-world problem into a usable equation is called mathematical modeling. The resulting equation along with a description of the variables involved is referred to as a mathematical model.

Example

Suppose that we wish to fill in a low swampy area with 120 cubic yards of dirt. Suppose that we will use a truck which is capable of moving 12 cubic yards of soil per load and that we can deliver 1 load of soil per hour around the clock until the job is complete. The amount of soil moved after t hours is given by the following chart.

Elapsed time (hours)	Amount of Soil
1	12
2	24
4	48
6	72
8	96
10	120

Display this data graphically. Write a model for the amount of soil moved in terms of elapsed hours.

Definition

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Example

If the price of gas is fixed at \$3.93 per gallon and g is the number of gallons that we wish to purchase, then the cost C will be a function of g, because for a given value of g, say $g = 10$ gallons there is only one possible cost that we would expect to pay, namely \$39.30.

We might represent the function of the previous example by

- The chart.

gallons	Cost
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- A graph (draw the graph).
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The graph in this example is an example of a continuous graph because it can be drawn without lifting our pencil.

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- Algebraic: If we have an equation relating the input variable to the output variable, we simply substitute the input for the input variable and calculate the value of the output variable.

Note (Vertical Line Test)

Suppose that we have a graph of data which are related in some way. If we can draw a vertical line which intersects the graph in 2 or more places then the variable whose values are plotted along the y-axis **cannot** be expressed as a function of the variable whose values are plotted along the x-axis.

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- Model: Always refers to an equation together with a description of the variables involved in the equation including units of measure.
- Equation: refers to a mathematical formula in one or more variables.
- Function: A rule which assigns to each possible input a unique output.

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If we are given the equation $y^2 - x = 2$, we can express x as a function of y but not the other way around.

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Describe the input and output variables. Be sure to discuss the units of measure involved.

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Describe the input and output variables. Be sure to discuss the units of measure involved.
What was the land value at the end of 2002?
When was the land worth $ 18,000?

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Business Term	Description
Fixed Costs	Costs that occur regardless of the number of items produces
Variable Costs	Productions costs
Total Costs	Fixed Cost + Variable Costs
Average Cost	$\frac{\text{total cost}}{\#\text{ items produced}}$
Revenue	Amount of income the business recieves
	Revenue = Profit + Total Cost
	Revenue = Price $\times$ Demand
Cost	Same as total cost
Profit	Profit = Revenue - Cost
Break-even point	Point where Cost = Revenue

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- The outputs of the two functions have the same units AND if there is a way to describe the resulting output if the functions are added or subtracted.
- In many examples the given functions may fail one of the above criteria, but can be adjusted so that the criteria are satisfied.

Example

Consider the total cost of a dairy business during a certain month. The fixed costs for the dairy for that month are \$ 21,000 and the variable cost incurred on day d of the month in question is given by $v(d) = -0.3d^2 + 6d + 250$ where $0 \leq d \leq 30$. Express the total cost of operations on day d of the month.

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- The output units for the 2 functions are well defined when they are multiplied or divided.
- There is a meaningful description of the resulting output when the functions have been multiplied or divided.

Example

Again consider the dairy business from before. Suppose that the price of milk on day d is given by $M(d) = 0.019d + 1.85$ dollars per gallon $(0 \leq d \leq 30)$ and that $S(d) = 1.7 + 0.4(0.82^d)$ thousand gallons of milk were sold on day d.

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Draw an input/output diagram for the multiplication function.
Write a function for daily revenue from milk sales?

Example

Continuing our example from before, suppose that we know that the total production on day d was $Q(d) = -1.5d^2 + 32d + 1803$ gallons of milk and that the total cost incurred on day d was $C(d) = -0.31d^2 + 6.2d + 1035$ dollars where again $0 \leq d \leq 30$.

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Write a function for the profit for the month if g gallons are produced and graph this function.

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Graph $K(g)$ and $T(g)$.
Write a function for the profit for the month if g gallons are produced and graph this function.
How much milk needs to be produced for the dairy to break even for the month?

Definition

Given two functions f and g we can form the composition function $g \circ f(x) = g(f(x))$ if the outputs of f can always be used as inputs for g. It must be the case also that the unit measure of the outputs of f are identical to the input units of g.

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Suppose that $C(p) = \sqrt{p}$ parts per million and that $p(t) = 0.4t^2 + 2.5$ thousand people t years after 1980 ($t \geq 0$).
Write a function that gives the lake contamination as a function of time.